

Real Option Valuation with Stochastic Interest Rate and Stochastic Volatility

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Abstract. Real options are one of the most interesting research topics in Finance since 1977 Stewart C. Myers from MIT Sloan School of Management published his pioneering article on this subject in the Journal of Financial Economics. Real options are techniques for supporting capital budgeting decisions that adapt techniques developed for financial securities options. The purpose of using this real option is to capture the options contained in projects that cannot be captured by the discounted cash flow model which operates as a basic framework for almost all financial analyzes. The process of valuing real options will be complemented by the stochastic interest rate and stochastic volatility to better capture the flexibility and volatility of the existing economic and financial situation. The valuation will use a Monte Carlo simulation with the MATLAB programming language on crude oil data from the North Sea oil field. Data were obtained from the thesis of Charlie Grafström and Leo Lundquist with the title "Real Option Valuation vs. DCF Evaluation – An Application to a North Sea oilfield".

Keyword: real options, stochastic interest rate model, stochastic volatility model, simulation

1. Introduction

Real options are one of the most interesting research topics in finance today. There are a lot of reasons for that. One of them is the weakness of the discounted cash flow model, which is the most frequently used valuation tool in finance. For example, the result of the net present value from the discounted cash flow model cannot capture the value of the option to delay, expand or leave the project. In addition, the current situation of the world economy which is very volatile and more flexible requires valuation tools that are better able to accommodate this volatility and flexibility and model it more accurately.

On the other hand, a manager in a company is needed to make decisions that can provide benefits and enlarge existing assets. This, of course, can happen one of which is to run a good project. For example, every year oil companies offer hundreds of millions of dollars to contract offshore oil mines, which are auctioned by the government. Providing accurate price estimates for these contracts is very important, both for the government and for the bidding companies. Ignoring flexibility in the petroleum business can cause undervaluation and misallocation of resources in the economy.

Therefore, in this study, we will discuss the valuation of real options with stochastic interest rates and stochastic volatility. First, we will discuss some of the material related to this study:

1.1. Stochastic Interest Rate Model

The first model for the structure of interest rates was the Vasicek model, which was developed by Oldrich Vasicek in 1977 [1]. The Vasicek model is a generalization of the Ornstein-Uhlenbeck process [2]:

$$dX_t = -\alpha X_t dt + \sigma dB_t \quad \dots (1)$$

with $t \geq 0, X_0 = x_0 \in \mathbb{R}$ is constan, $\alpha \in \mathbb{R}^+, \sigma \in \mathbb{R}^2$, and dB_t is the Wiener process.

Then introduce a constant drift term β . Short-rate in the Vasicek model is given by stochastic differential equations:

$$dr_t = \alpha \left(\frac{\beta}{\alpha} - r_t \right) dt + \sigma dB_t \quad \dots (2)$$

with $r_0 \in I = \mathbb{R}$, $\alpha \in \mathbb{R}^+$, $\beta \in \mathbb{R}$, and $\sigma \in \mathbb{R}^+$ are constant. The parameter α is called mean reversion force whereas α/β is called mean reversion level. For $t \rightarrow \infty$ the stationary distribution in the Vasicek model is $N\left(\frac{\beta}{\alpha}, \frac{\sigma^2}{2\alpha}\right)$. In case the $\beta = 0$, Vasicek model will return to an Ornstein-Uhlenbeck process [3].

1.2. Stochastic Volatility Model

Volatility cannot be observed directly because it is not traded. However, from empirical studies, volatility can be estimated from returns derived from basic assets [4]. The following is the stochastic volatility model equation [5]:

$$dv(t) = \kappa[\theta - v(t)]dt + \sigma\sqrt{v(t)}dZ_2(t) \quad \dots (3)$$

with $\mathcal{U}(0)$, θ , κ , and σ are a constant and the Wiener $Z_2(t)$ process has a correlation ρ with the Wiener process. In this model, $v(t)$ is a non-central chi-square distribution

1.3. Stochastic Price Model

The stochastic price forecasting model to be used in modeling crude oil prices in this research is the reverting commodity prices model used in several previous journals (Smith and McCardle, 1998; Baker, Mayfield and Parsons, 1998; Salahor 1998) and summarized by Samis, Poulin and Blais (2005) [6]. This model uses the standard GBM (Geometric Brown Motion) formula for continuous spot-price changes (dS), including reversion to the mean and error factors as represented by Samis, Poulin and Blais (2005) [6]:

$$dS = \left[\alpha^* + \frac{1}{2}\sigma^2 - \gamma \ln \frac{S}{S^*} \right] Sdt + \sigma SdZ \quad \dots (4)$$

with α^* short term rate, S present price, S^* current long-term median price, σ short-term volatility, γ mean reversion force, and Z Wiener process.

1.4. Present Value of Developed Reserved Project

The first thing to do to calculate the real option value is to determine the present value of the developed reserve of the project. The present value of the developed reserve of the oil mining project can be determined by the certainty equivalent (CEQ) approach [7]. More specifically, the calculation of the cash flow that enters when the project is running is done to estimate the present value of the project. It is assumed that all the production will be according to plan when executed and there is no depreciation value ($D_t = 0$). The CEQ value of a developed project is [7]:

$$NPV_{CEQ} = -k + \sum_{t=T}^N e^{-rt} [(Q_t S_t - C_t X_t)(1 - h_t) + h_t D_t] \quad \dots (5)$$

$$PV_{CEQ} = \sum_{t=T}^N e^{-rt} [(Q_t S_t - C_t X_t)(1 - h_t) + h_t D_t] \quad \dots (6)$$

with k initial investment, N production end time, r risk-free interest rate, Q_t production in year t , C_t expenditure in year t , S_t crude oil price in year t , h_t corporate tax rate in year t , X_t forward exchange rate in year t , and D_t depreciation in year t .

2. Experimental Method

Determination of real option values in this research will be executed by Monte Carlo simulation on crude oil data taken from the US Energy Information Administration [8] in four stages, that is in January 1995-December 2000 which assumed prices were in stable condition, then January 2001- December 2006 which assumed prices to rise, January 2007-July 2013 which assumed prices were unstable, and from January 1995-July 2013 which stated all data. In real options valuation, the input parameters needed are almost the same as the valuation of options in finance. As shown in table 1.

Table 1
Parameters on Options in Finance vs. Real Options

Options in Finance	Real Options
option value in finance	value of undeveloped reserve (V)
present stock price	present value of developed reserve (S)
exercise price	initial investment to run reserves (K)
dividend yield	net convenience yield (y)
risk free interest rates	risk free interest rates (r)
stock volatility	volatility of developed reserve (σ)
maturity time of options	maturity time of investment rights (T)

Then the assumptions of production data and oil project expenditure are taken from the thesis of Charlie Grafström and Leo Lundquist [7] to calculate CEQ and real options value. This thesis explains about real options valuation with two factors developed by Gibson (1990) by using the finite difference method on the North Sea oil mining investments that have not been implemented in 2002. The following is the production and expenditure data:

Table 2
Production and Expenditure Data

Year	Production	Expenditure
1	-	-132,662,000.00
2	-	-
3	-	-
4	7,793,487.00	-25,542,000.00
5	3,312,984.00	-12,834,000.00
6	1,988,994.00	-9,129,000.00
7	1,489,489.00	-7,761,000.00
8	1,191,591.00	-6,992,000.00
9	1,074,237.00	-6,723,000.00
10	965,911.00	-6,491,000.00
11	869,621.00	-6,308,000.00
12	625,886.00	-6,138,000.00
13	156,472.00	-9,133,000.00

The maximum likelihood estimation will be used to estimate the parameters of the stochastic interest rate model, the stochastic volatility model and the parameters for the stochastic crude oil price forecasting model. First, the stochastic differential equation for Ornstein Uhlenbeck is given by [2]:

$$dS_t = \lambda(\mu - S_t)dt + \sigma dW_t \quad \dots (7)$$

with λ is mean reversion force, μ is mean reversion level, and σ is volatility of S .

The conditional probability density function can be derived by combining the equation (7) with the probability density function of the normal distribution function. The equation of the conditional probability density of an observation S_{i+1} is given by previous observations of S_i (with a step between them δ) given by:

$$f(S_{i+1}|S_i; \mu, \lambda, \hat{\sigma}) = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp \left[-\frac{(S_i - S_{i-1}e^{-\lambda\delta} - \mu(1 - e^{-\lambda\delta}))^2}{2\hat{\sigma}^2} \right] \quad \dots (8)$$

with,

$$\hat{\sigma}^2 = \sigma^2 (1 - e^{-2\lambda\delta})/2\lambda \quad \dots (9)$$

The log-likelihood function for the observation set $(S_0, S_1, S_2, \dots, S_n)$ can be derived from the probability density function:

$$l(\mu, \lambda, \hat{\sigma}) = -\frac{n}{2} \ln(2\pi) - n \ln(\hat{\sigma}) - \frac{1}{2\hat{\sigma}^2} \sum_{i=1}^n (S_i - S_{i-1}e^{-\lambda\delta} - \mu(1 - e^{-\lambda\delta}))^2 \quad \dots (10)$$

the parameters μ, λ , and σ will be estimated by MLE. To find σ can be solved by substituting λ to μ . From there we get the solution of equation [2]:

$$\mu = \frac{S_y S_{xx} - S_x S_{xy}}{n(S_{xx} S_{yy}) - (S_x^2 - S_x S_y)} \quad \dots (11)$$

$$\lambda = -\frac{1}{\delta} \ln \left[\frac{S_{xy} - \mu S_x - \mu S_y + n\mu^2}{S_{xx} - 2\mu S_x + n\mu^2} \right] \quad \dots (12)$$

$$\hat{\sigma}^2 = \frac{1}{n} [S_{yy} - 2\alpha S_{xy} + \alpha^2 S_{xx} - 2\mu(1 - \alpha)(S_y - \alpha S_x) + n\mu^2(1 - \alpha)^2]$$

$$\sigma^2 = \hat{\sigma}^2 \frac{2\lambda}{1 - \alpha^2} \quad \dots (13)$$

with

$$\alpha = e^{-\lambda\delta}; S_x = \sum_{i=1}^n S_{i-1}; S_y = \sum_{i=1}^n S_i; S_{xx} = \sum_{i=1}^n S_{i-1}^2; S_{xy} = \sum_{i=1}^n S_{i-1} S_i; S_{yy} = \sum_{i=1}^n S_i^2 \quad (14)$$

2.1. Step of Real Option Valuation with Monte Carlo Simulation

According to [9], the American option is calculated by comparing the profit if the option is exercised and the discount value of the expected value if the option is held for the next time, for each time before maturity $t < T$. Monte Carlo simulations can be used to evaluate American options by first identifying an optimal boundary value for implementation, which is also called locus of critical prices $\bar{S}_t^*(S_t)$. At each time t , for each initial value S_t there is \bar{S}_t^* where for the call option, the smallest value

of \bar{S}_t will be result only one $\bar{S}_t^*(S_t)$. Following are the steps to determine the real option value with Monte Carlo simulation:

1. Simulate M lines of crude oil prices with the simulation start time of price t_g for time $t = t_g, \dots, t_{n-1} = T - \tau$
2. For each time t , use the locus of critical prices for $S_{t,i}$ located between $S_t(j)$ and $S_t(j + 1)$ for estimating the critical value \bar{S}_t^* by using cubic spline interpolation type clamped spline. For each iteration $t < T$ if $\bar{S}_{t,i} > \bar{S}_t^*(S_t)$, option value at time t :
- 3.

$$C_i(S_t) = \bar{S}_{t,i} - K \quad \dots (15)$$

for $t = T$

$$C_i(S_t) = \max\{(\bar{S}_{t,i} - K), 0\} \quad \dots (16)$$

then discounted to $t = 0$

$$C_{i,0} = e^{-rt\tau} C_i(S_t) \quad \dots (17)$$

- 4.
5. Real option value
- 6.

$$C_0 = \frac{1}{M} \sum_{i=1}^M C_{i,0} \quad \dots (18)$$

with a confidence interval of 95%

$$\left[C_0 - 1.96 \frac{b_M}{\sqrt{M}}, C_0 + 1.96 \frac{b_M}{\sqrt{M}} \right] \quad \dots (19)$$

3. Result and Discussion

Table 3 shows the parameters in stochastic modeling interest rates, volatility, and crude oil prices used in this research by using the maximum likelihood estimation in equation (10). In the likelihood estimation here, first, the historical data is entered from the parameters to be estimated, then determine δ , which is the time of observation of data in years. Historical interest rate data are taken from the Board of Governors of the Federal Reserve System [10] and historical volatility data are taken from the return of crude oil spot price. Next find S_x, S_y, S_{xx}, S_{xy} , and S_{yy} as in equation (14) which will be substituted to equations (11), (12), and (13) to find the mean reversion level μ , mean reversion force λ , and volatility σ . The following are the results:

Table 3
Real Option Valuation Parameters

Parameter	Data Stage				
	Jan 05 - Des 00	Jan 01 - Des 06	Jan 07 - Jul 13	Jan 05 - Jul 13	
Interest Rate	mean reversion level	0.0574	0.0451	0.0269	0.0366
	mean reversion force	1.3106	1.7392	0.6815	0.3037
	volatility	0.0085	0.0090	0.0101	0.0092
Volatility	mean reversion level	0.3325	0.2680	0.3136	0.3383
	mean reversion force	1.5078	1.3454	1.1724	2.9961
	volatility	0.1580	0.1778	0.2717	0.3442
Initial Parameter	S_0	26.8	61.05	105.03	105.03
	S^*	22.34	58.307	87.89	100
	r_0	0.0513	0.0503	0.0252	0.0252
	v_0	0.3775	0.2755	0.2295	0.2295

In the initial parameter in table 3, the initial crude oil price S_0 , the initial interest rate r_0 , and the initial volatility v_0 are taken from historical data at the end of each period and long term crude oil S^* is taken from the mean reversion level of crude oil data per period.

The simulation is executed with two conditions, the first condition of the simulation of valuation of real option values is executed with constant interest rates and constant volatility with the simulation results shown in table 4 at section constant. In this section, the parameter of interest rate r is taken from the mean reversion level in table 4 as a constant value. Then the second condition of the simulation of valuation of real option values is executed with stochastic interest rates and stochastic volatility with the simulation results shown in table 4 at section stochastic. Parameters of mean reversion level, mean reversion force, and volatility to model stochastic interest rates and stochastic volatility are given in table 3. Then further, the Monte Carlo simulation will be execute with 7000 steps/repetition. Here is the results:

Table 4
Results of Real Options Valuation

Data Stage	Repetition	Interest Rate and Volatility	Real Option Value	Difference (95%)
'95 – '00	7.000	Constant	168,615,810.70	15,438,257.68
		Stochastic	171,215,000.61	10,353,857.63
'01 – '06		Constant	534,690,315.05	21,965,857.62
		Stochastic	535,384,264.76	20,974,480.96
'07 – '13		Constant	807,499,124.51	40,539,968.38
		Stochastic	820,150,134.94	37,761,652.50
'95 – '13	Constant	936,088,461.56	51,380,637.17	
	Stochastic	942,852,379.97	44,038,933.12	

From the various possible results that can be obtained during the simulation, table 4.8 shows that the real option value will be higher when the price of crude oil is higher, after that the real option value with stochastic interest rate and stochastic volatility is higher than the simulation with constant interest rates and constant volatility, and the last is the difference in the confidence interval for the simulation with a stochastic interest rate and stochastic volatility is smaller than the simulation with constant interest rates and constant volatility.

The real option value will be higher when the higher crude oil prices are exemplified by the real option value in the period 1995-2000, with the long-term crude oil price S^* 22.34, the simulation result of real option is 168,615,810.70. Whereas in the period 2001-2005 with the price of long-term crude oil S^* 58.307, the simulation result of real option is 534,690,315.05. Last one, the simulation result of real option is 807,499,124.51 in the period 2007-2013 with the price of long-term crude oil S^* 87.89 (see table 3).

The real option value with stochastic interest rate and stochastic volatility is higher than the simulation with constant interest rates and constant volatility are exemplified in the period 1995-2000 when the simulation result of real options with stochastic interest rates and stochastic volatility is 171,215,000.62 while the simulation result of real options with constant interest rates and constant volatility is 168,615,810.71. Different by 2,599,189.91.

Last, the difference in the confidence interval for the simulation with a stochastic interest rate and stochastic volatility is smaller than the simulation with constant interest rates and constant volatility are exemplified in the period 1995-2000 when the difference in the confidence interval of real options value with stochastic interest rates and stochastic volatility is 10,353,857.63 while the difference in the confidence interval of real options value with constant interest rates and constant volatility is 15,438,257.69. Different by 5,084,400.06.

4. Conclusion

The result of real option value will be higher when the higher crude oil prices and the result with Monte Carlo simulation will be better when the simulation using higher repetition, both for simulation with constant interest rate and constant volatility or for stochastic interest rate and stochastic volatility. Furthermore, from the various possible results that can be obtained during the simulation with a large number of steps/repetitions, the real option value with stochastic interest rate and stochastic volatility is higher with a smaller difference in the confidence interval than the simulation with constant interest rates and constant volatility.

Acknowledgements

The Author thanks to the Direktorat Riset dan Pengabdian Masyarakat (DPRM), Direktorat Jenderal Penguatan Riset dan Pembangunan, Kementerian Riset, Teknologi, dan Pendidikan tinggi for supporting and funding this research. We would like to express a lot of thanks for this support.

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