Statistika, Vol. 10 No. 1, 35 – 41
Mei 2010

Updating Reservoir Models Using Ensemble Kalman Filter

SUTAWANIR DARWIS[1], AGUS YODI GUNAWAN[2], SRI WAHYUNINGSIH[1,3], NURTITI SUNUSI[1,4], ACENG KOMARUDIN MUTAQIN[1,5], NINA FITRIYATI[1]

[1]Statistics Research Division, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia.
[2]Industrial and Financial Mathematics Research Division, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia.
[3]Statistics Study Program, Faculty of Mathematics and Natural Sciences, Universitas Mulawarman, Indonesia.
[4]Mathematics Department, Faculty of Mathematics and Natural Sciences, Universitas Haluoleo, Indonesia.
[5]Statistics Department, Faculty of Mathematics and Natural Sciences, Universitas Islam Bandung, Indonesia.
E-mail: sdarwis@math.itb.ac.id

ABSTRACT

The Ensemble Kalman Filter (EnKF) has gain popularity as a methodology for real time updates of reservoir models. A sample of models is updated whenever observation data available. Successful application of EnKF to estimate reservoir properties has been reported. A flow modeling is missing in this research area. This paper presents the applicability of EnKF in flow modeling for three cases: infinite reservoir, bounded reservoir and one dimensional composite reservoir. The solution of flow equation was derived and used as a modeling component of state space modeling of Kalman filter updating formula. This three reservoir models shows that the EnKF methodology can be used for updating the reservoir models.
Keywords: reservoir flow modeling; Kalman filter; ensemble Kalman filter.

1. INTRODUCTION

The Ensemble Kalman Filter (EnKF) is a promising method for updating reservoir properties of reservoir simulation models (Naevdal et al., 2007). Darcy's law, mass conservation, petro physical behavior are the basics for modeling the fluid flow in the reservoir. The use of EnKF for updating reservoir models from the pressure measurements has been a topic for research. The EnKF can update static (permeability, porosity), and dynamic (pressure, gas oil ratio). The prior and posterior step are run sequentially. The flow simulations end at the next point in time where new measurements are to assimilated. The reservoir simulator is run once for each member of the ensemble. Applying the simulator on the state is used to estimate the covariance. In the posterior step, each member of the ensemble is updated. In the prior step, only dynamic variables are updated, the static are updated in the posterior step. The EnKF is able to do continuous updating of the reservoir model, the reservoir models are kept up to date (Almendral-Vazquez, and Syversveen, 2006). Using reservoir simulator, Jafarpour and McLaughlin (2007) demonstrated the application of EnKF in water flooding experiments. However, the applicability of EnKF methodology coupled with reservoir flow modeling has not been investigated. This paper presents the applicability of EnKF in flow modeling for three cases: infinite reservoir, bounded reservoir and one dimensional composite reservoir. The solution of flow equation was derived and used as a modeling component of state space modeling of Kalman filter updating formula. This three reservoir models shows that the EnKF methodology can be used for updating the reservoir models.

2. METHODOLOGY

The diffusivity equation is considered as one most important expression in flow modeling. The model is derived under assumption that the permeability and viscosity are constant over pressure, time and distance, and the fluid is assumed slightly compressible. The parameters are: P (pressure in psi), r (radius in ft), t (time in hours), k (permeability in mD), μ (viscosity in cp), ϕ (porosity in %), c (total compressibility in psi^{-1}). The equation is defined in a cylindrical reservoir with a hole as the well. Figure 1 shows a slice of the reservoir with geometry parameters: r_w (well radius), r_e (external radius), $P = P_{BHP}$ (bottom hole pressure), and h (thickness of the reservoir).

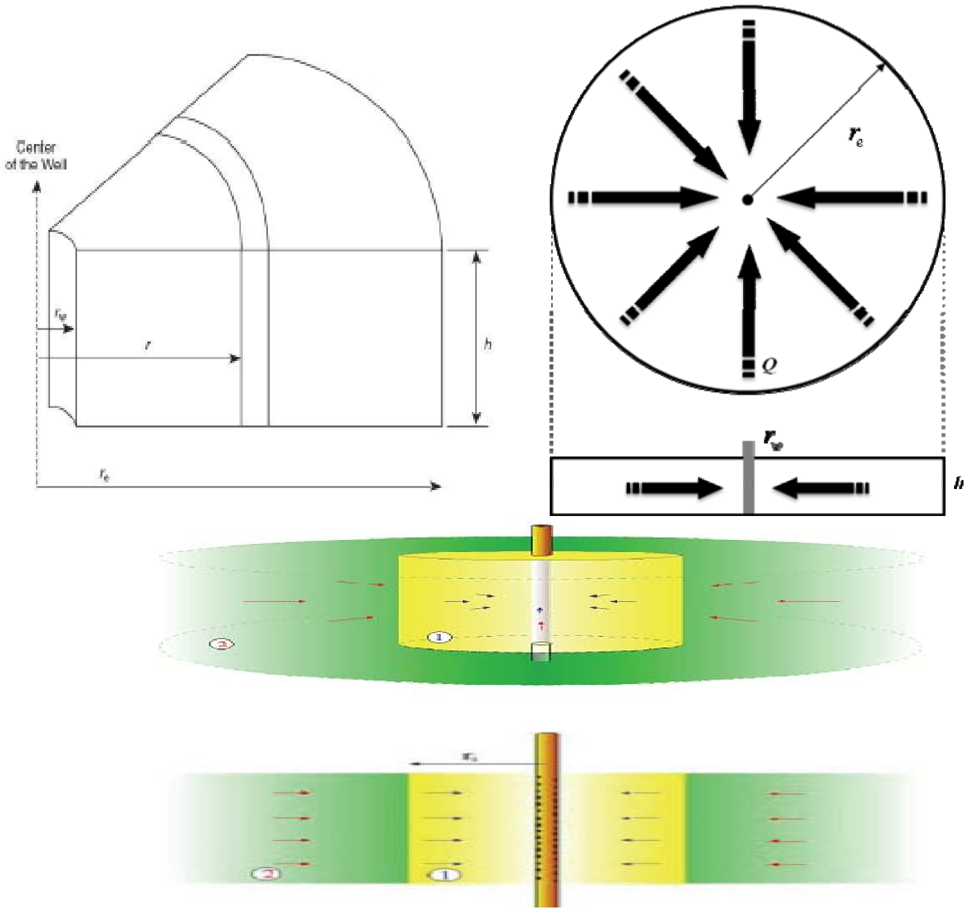

Figure 1. Reservoir model: A slice of a bounded reservoir geometry (Almendral-Vazquez, Syversveen, 2006), r_w is the well radius, r_e is the reservoir radius, and h is the reservoir thickness. The reservoir is modeled as a cylinder with a hole at the center.

Infinite reservoir. Radial flow (reservoir infinite acting) into a well, the pressure P depends on the radius r and time t

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c}{.0002637\ k}\frac{\partial P}{\partial t}$$

$$P(r,0) = P_i, P(\infty, t) = P_i, \quad .001127\frac{2\pi rhk}{\mu}\frac{\partial P}{\partial r}\Big|_{r=r_w} = q$$

Using Hankel transformation, the solution is given by the expression

$$P(r,t) = P_i - \left(\frac{70.61 Q_0 \mu B_0}{kh}\right) Ei\left(\frac{948\phi c r^2}{kt}\right)$$

B_0 is the oil formation volume (well fields barrels/stock tank barrels), Q_0 is the oil flow, $B_0 Q_0 = q$ is the flow rate, Ei is the exponential integral $Ei(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt$

Bounded reservoir. Consider a bounded reservoir model with inner and outer boundary: constant rate inner boundary and no flow outer boundary, in dimensionless notation

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D \frac{\partial P_D}{\partial r_D}\right) = \frac{\partial P_D}{\partial t_D},\ P(r_D,0) = 0,\ r_D \frac{\partial P_D}{\partial r_D}|_{r_D=1} = -1,\ \frac{\partial P_D}{\partial r_D}|_{r_D=r_{eD}} = 0$$

Using Laplace transform, the solution is given by

$$P(r_w,t) = P_i - \frac{Q\mu}{2\pi kh}\left[2\frac{t_D}{r_{eD}^2} + \ln\ r_{eD} - \frac{3}{4} + 2\sum_{n=1}^{\infty}\frac{1}{\alpha_n^2}\left[\frac{J_1^2(\alpha_n r_{eD})}{J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)}\right]e^{-\alpha_n^2 t_D}\right]$$

with α_n is the root of $J_1(\alpha_n r_{eD}) Y_1(\alpha_n) - J_1(\alpha n) Y_1(\alpha_n r_{eD}) = 0$, the dimensionless parameters are $r_D = r / r_w$, $P_D = \frac{P_i - P}{P_{ch}}$, $P_{ch} = \frac{141.2 qB\mu}{kh}$, $t_D = t / t_{ch}$, $t_{ch} = \frac{\phi\mu c r_w^2}{.0002637k}$.

Composite reservoir. Radial fluid flow through composite reservoir regions represented by the diffusivity equations:

$$\frac{\partial P_1}{\partial r^2} + \frac{1}{r}\frac{\partial P_1}{\partial r} = \left(\frac{\phi\mu c}{k}\right)_1 \frac{\partial P_1}{\partial t} \quad r_w < r < R$$

$$\frac{\partial P_2}{\partial r^2} + \frac{1}{r}\frac{\partial P_2}{\partial r} = \left(\frac{\phi\mu c}{k}\right)_2 \frac{\partial P_2}{\partial t} \quad r \geq R$$

$$P_1(r,0) = P_i, P_2(r,0) = P_i, P_1(r_w,t) = P_w$$

$$P_1(R,t) = P_2(R,t), \frac{\partial P_2}{\partial r}|_{r=R} = \lambda \frac{\partial P_1}{\partial r}|_{r=R}, \lambda = \left(\frac{k}{\mu}\right)_1 \left(\frac{k}{\mu}\right)_2$$

$$P_2(\infty,t) = P_i$$

One dimensional composite reservoir. Consider a semi infinite composite regions $(-\ell,0)\bigcup(0,\infty)$, the diffusivity equations are

$$\frac{\partial^2 P_1}{\partial x^2} = \frac{1}{k_1}\frac{\partial P_1}{\partial t} \quad -\ell<x<0 \qquad \frac{\partial^2 P_2}{\partial x^2} = \frac{1}{k_2}\frac{\partial P_2}{\partial t}, \quad x \geq 0$$

$$k_1 \frac{\partial P_1}{\partial x}|_{x=0} = k_2 \frac{\partial P_2}{\partial x}|_{x=0}, \quad P_1(0,t) = P_2(0,t)$$

$$P_1(x,0) = 0, P_2(x,0) = 0, P_1(-\ell,0) = P$$

The solutions are given by (Carslaw, Jaeger, 1980: 319)

$$P_1(x,t) = P\sum_{i=0}^{\infty}\alpha^i \left\{ \begin{array}{l} \operatorname{erfc}\dfrac{(2i-1)\ell + x}{2\sqrt{k_1 t}} - \\ \alpha \ \operatorname{erfc}\dfrac{(2i+1)\ell - x}{2\sqrt{k_1 t}} \end{array} \right\}$$

$$P_2(x,t) = \frac{2P}{1+\sigma}\sum_{i=0}^{\infty}\alpha^i \ \operatorname{erfc}\frac{(2i+1)\ell + kx}{2\sqrt{k_1 t}}$$

$$k = \sqrt{\frac{k_1}{k_2}}, \quad \sigma = \frac{k_2}{k_1}k, \quad \alpha = \frac{\sigma-1}{\sigma+1}$$

Consider a state-space model $X_t = f(X_{t-1}) + \varepsilon_t^m$, $Y_t = GX_t + \varepsilon_t^o$ where X_t is the state at time t, ε^m is the model error , $\varepsilon^o = \varepsilon^{obs}$ is the observation error, f is a nonlinear function of the state X. Sakov and Oke (2008) derived a deterministic formulation of the EnKF. The methodology is based on prior-posterior equation

$$X^a = X^f + K\left(Y - GX^f\right), \ K = P^f G^t \left(GP^f G^t + R\right)^{-1}$$

where K is the Kalman gain, X^f is the prior, X^a is the posterior, Y is the observations, G is the observation matrix, P is the forecast error covariance matrix, R is the observation error covariance matrix. The covariance P is manipulated via a sample of states $X = (X_1, \ldots, X_m)$, where m is the sample (ensemble) size

$$P = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_i - \bar{X}\right)\left(X_i - \bar{X}\right)^t = \frac{1}{m-1}AA^t$$

where $\bar{X} = \frac{1}{m}\sum_{i=1}^{m} X_i$ is the sample mean, and $A = (A_1, \ldots, A_m)$ is the sampling residuals, $A_i = X_i - \bar{X}$. The EnKF processes each sample member using the posterior equation

$$X_i^a = X_i^f + K\left(Y + \varepsilon_i^m - GX_i^f\right), i = 1, \ldots, m$$

where ε_i^m is an error of observations Y, $\sum_{1}^{m}\varepsilon_i^m = 0$. The equation for the posterior of sampling error $A_i^a = A_i^f + K\left(\varepsilon_i^m - GA_i^f\right), i = 1, \ldots, m$ or in a matrix form $A^a = A^f + K\left(\varepsilon - GA^f\right)$.

The posterior error covariance is (Sakov and Oke, 2008, 362)

$$\begin{aligned} P^a &= \frac{1}{m-1}A^a A^{at} \\ &= P^f - P^f G^t K^t - KGP^f + KGP^f G^t K^t \\ &+ \frac{1}{m-1}K\varepsilon\varepsilon^t K^t + \frac{1}{m-1}(I - KG)A^f\varepsilon^t K^t \\ &+ \frac{1}{m-1}K\varepsilon A^{ft}\left(I - G^t K^t\right) \end{aligned}$$

3. RESULTS AND DISCUSSION

This section intend to show how the EnKF may be used to update the reservoir model and to recover the true parameter values. The reservoir model is constant flow rate infinite (transient) reservoir Figure 2 shows the results for estimating storativity $S = h\phi c$ and transmissivity $T = kh/\mu$. The experiment was run for time period T = 5000 hours. The observations correct the parameters values, and a significant reduction in standard deviation. The iteration for transmissivity converges after 500 hours, the iteration for storativity takes longer iterations to converge. The experiment shows that the method works properly in well testing and interference analysis using line source solution.

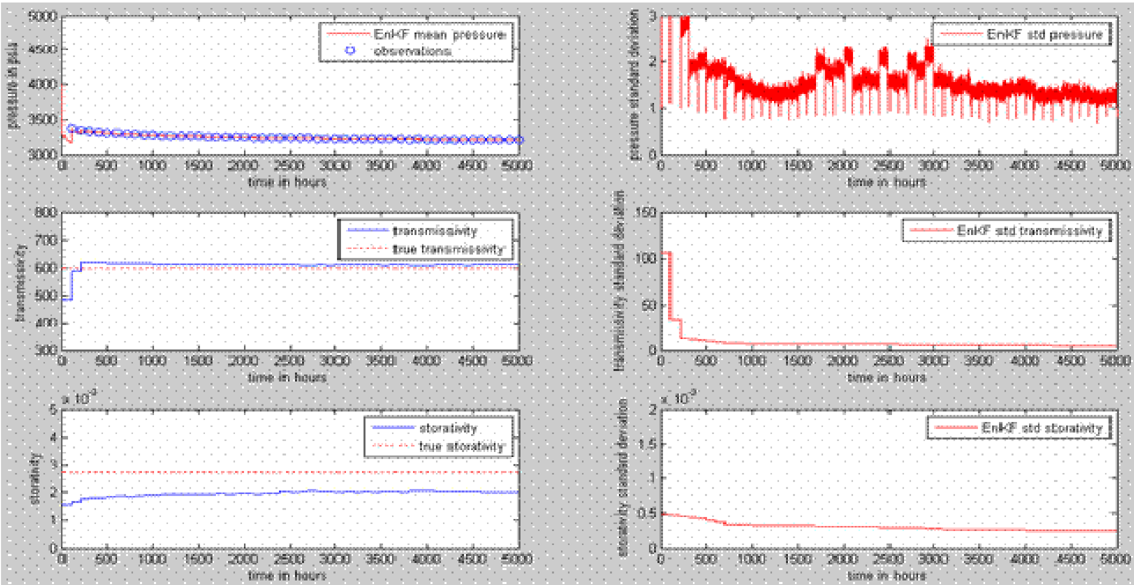

Figure 2. EnKF for interference test based on line source solution. The transmissivity $T = kh/\mu$ and storativity $S = h\phi c$ converge to the true values

Figure 3 shows 20 measurements (pressure) from build up test (Permadi, 2004) in time period of 72 hours. Due to the specific experiment, the pressures are increasing. The bounded constant flow rate and no flow is chosen as reservoir models. Using traditional approach, the permeability was estimated as $k_{true} = 7.664$ mD. Using EnKF, the objective is to show that the filter will update the reservoir model after a number of iterations. The initial permeabilities was generated from $N(11,3;1)$, the ensemble size is *m* = 100. The update of permeabilities converge to $k_{true} = 7.664$ mD. The simulated P_{WS} was obtained from analytical solution with parameters $P_i = 5000$ psi, time step 0.05 hours, experiment time 72 hours, number of observations 20. The simulated pressure converge to the observed pressure (Figure 4).

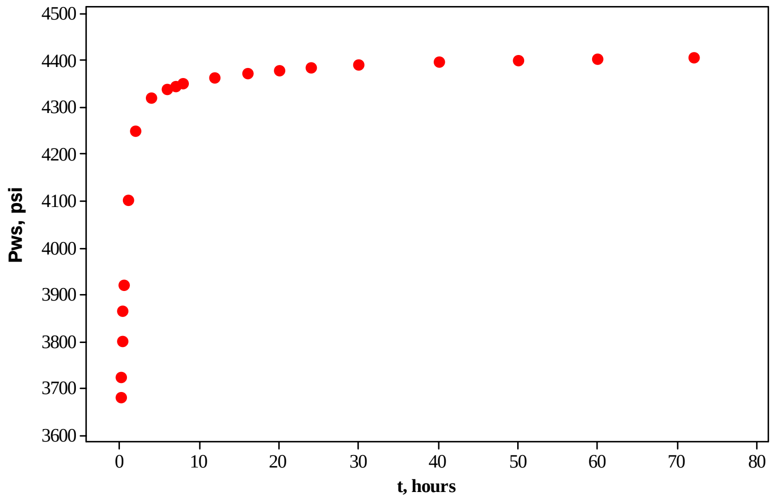

Figure 3. Pressure observations in build up test (Permadi, 2004); 72 hours after $t_s = 13630$ hours production. The pressures increase as function of time t. There are three time regions: Early Time Region (ETR), Middle TR (MTR), and Late TR (LTR).

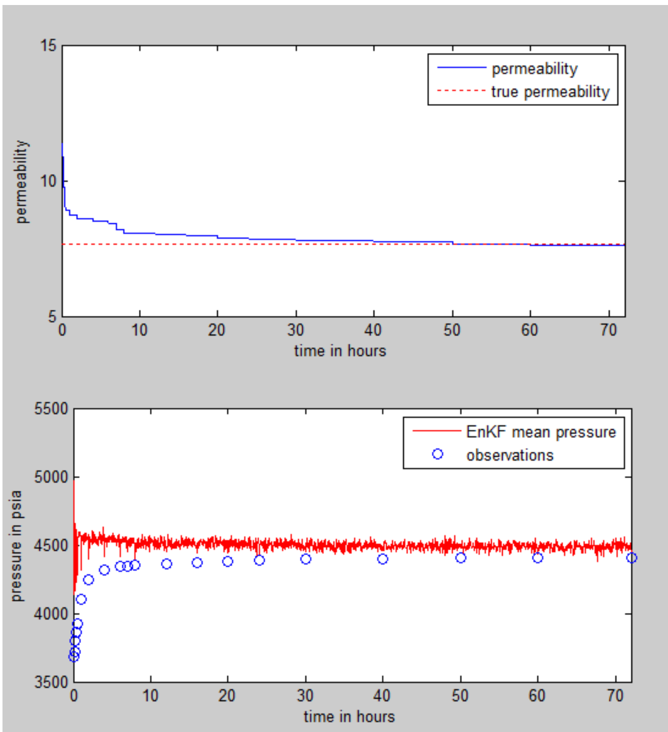

Figure 4. Bounded constant flow rate no flow homogeneous reservoir. The mean of initial permeability distribution is larger than the true permeability. The initial permeabilities was generated from $N(11,3;1)$, the ensemble size is m = 100. The update of permeabilities converge to $k_{true} = 7.664$ mD. The simulated P_{WS} was obtained from analytical solution with parameters $P_i = 5000$ psi, time step 0.05 hours, experiment time 72 hours, number of observations 20. The simulated pressure converge to the observed pressure.

The results from the third reservoir model; one dimensional composite reservoir, is shown in Figure 5. As expected, the permeabilities converge to the true permeabilities, and the simulated pressure matched the observed pressures. For the case of composite reservoir, the EnKF methodology can be used to recover the true reservoir model.

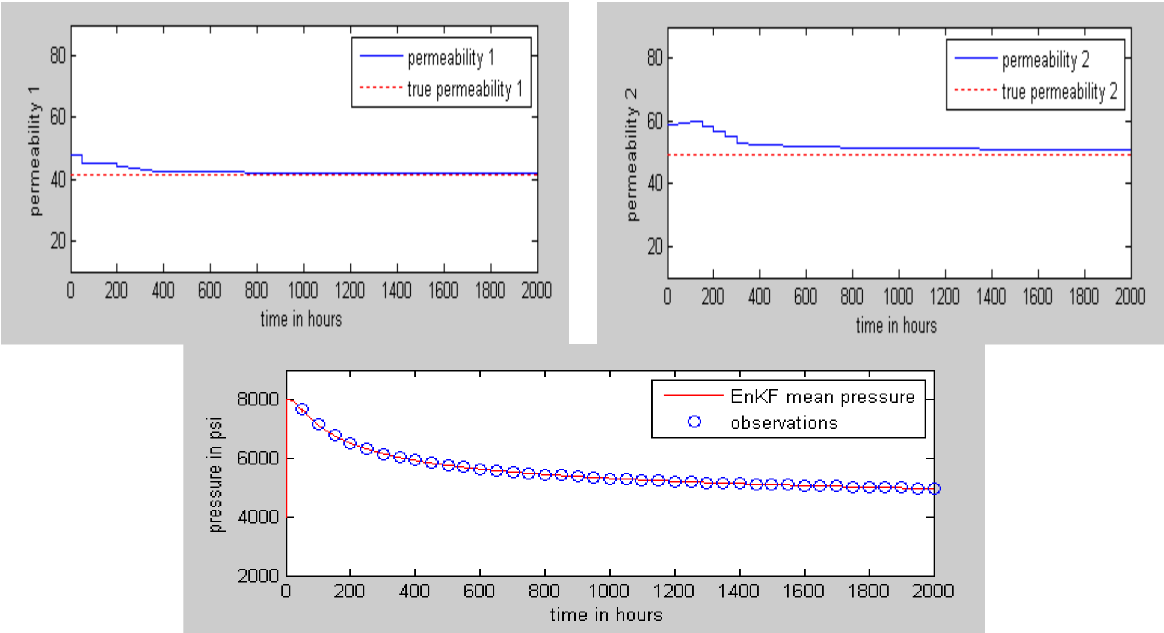

Figure 5. Composite reservoir. The evolution for permeabilities of region I and region II and pressure (simulated and observed) $\ell = 200, x = 100$, $\mu = 1.5$ cp, $\phi = 15\%$, $P_i = 4000$ psi, $c = 12\times10^{-6}\,\text{psi}^{-1}$.

4. SUMMARY AND CONCLUSIONS

In traditional history matching, the reservoir parameters are adjusted such that the flow simulations using the adjusted parameters match the measurements. Traditional methods do not allow for model updating as new measurements become available. EnKF is able to update the reservoir model in the posterior step. This paper demonstrated the applicability of the ensemble Kalman filter for updating reservoir models. The basics are the flow equation and the solution of the flow equation. Three models are considered: infinite reservoir (transient flow rate), radial bounded no flow homogeneous reservoir, and one dimensional composite reservoir. The solution of the flow equation is considered as reservoir model. The methodology can be adapted for commercial simulator. The updates are improved after assimilation the observation data. As expected, the iteration of permeability is converging to the true value.

REFERENCES

[1]. Almendral-Vazquez A, Syversveen A R, 2006, The Ensemble Kalman Filter – Theory and Application in Oil Industry, Norsk Regnesentral, Norwegian Computing Centre.

[2]. Jafarpour B, and McLaughlin, 2007, History Matching with an Ensemble Kalman Filter and Discrete Cosine Parametrization, SPE 108761 SPE Annual Technical Conference, Anaheim, 11-14 Nov 2007.

[3]. Naevdal G, Bianco A, Cominelli A, Dovera L, Lorentzen R J, and Valles B, 2007, State Estimation of A Large Scale System in Petroleum Industry: The Ensemble Kalman Filter For Updating Reservoir Models, 8th International IFAC Symposium on Dynamics and Control of Process Systems, Mexico.

[4]. Permadi A K, 2004, Reservoir Engineering, Institut Teknologi Bandung.

[5]. Sakov P and Oke P. R, 2008, A Deterministic Formulation of the Ensemble Kalman Filter: An Alternatif to Ensemble Square Root Filters, Tellus, 60A, 361-371.