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Measurement System Analysis Using Repeatability and Reproducibility Techniques

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ABSTRACT

To monitor and improve any manufacturing process in quality control, it is important to measure the cause of the attribute for that process. The quality of measurement data depends on the repeated measurements obtained through measurement systems which will operate at a given condition. This study uses a type of statistical quality control technique called Repeatability and Reproducibility (R&R) to determine how much of the process variation is caused by the variation of the measurement system that is being used. The two types of charts that were used for this study were the ANOVA chart and the Mean & Average chart.

1. Introduction

In engineering and manufacturing, quality control is a set of measurements taken to ensure that faulty products are not produced. Quality control includes all the procedures done and taken to fulfil the demand for quality products. To inspect and repair any manufacturing process, it is important to measure the output attributes. For any group of measurements gathered for this purpose, there will be at least some variation caused by the measurement system. Measurement system analysis is divided into two types of analysis: those made for qualitative and quantitative measurement data and those done with test materials. The same characteristic of the product should be measured repeatedly in order to identify the sensitivity of the measurement process.

The main purpose of a measurement system is to minimize measurement errors. So, it is important to identify the deviation sources that could affect the results produced by the system as well as to ensure that the number of the measurements is adequate enough both for deviation and for sensitivity. To ensure that the variation from the measurement system is not at critical level, it is important to conduct gage repeatability and reproducibility study on measurement system.

The most effective time to perform gage R&R studies is before the gage is used to gather data or inspect products. In this preventative mode, problems that are associated with taking data, charting, capability analysis, acceptance errors and so on are avoided because the measurement system that fails a gage R&R Study is corrected before data is collected.

Along with the prevention mode comes the task of performing a variety of studies before data collection can begin. Another school of thought is the reactive mode. In this mode, if control and capability problems arise that are unexplained in the process, one challenges the measurement system and performs a gage R&R study at that time. The reactive mode is not recommended because in most cases, a considerable amount of time and energy has been wasted on collecting bad data.

Gage R&R is a proven analysis method for measuring the capability of the measurement system and isolating the primary sources of measurement errors (observer or instrument). Using gage R&R, the intent is to (1) identify the measurement process that require improvement, (2) identify the source of measurement errors (observer or equipment), and (3) correct the errors.

Repeatability is the variability of the measurements obtained by one person while measuring the same item repeatedly. This is also known as the inherent precision of the measurement equipment. Reproducibility is the variability of the measurement system caused by differences in operator behaviour. Mathematically, it is the variability of the average values obtained by several operators while measuring the same item.

The most commonly used method for computing repeatability and reproducibility is the Range and Average method. The ANOVA method is more accurate but it was not so popular in the past because of the complex mathematics involved. However, with more advanced computer software, there is no excuse for not using the more accurate ANOVA method.

2. Range and Average Method

This method computes the total measurement system variability, and allows the total measurement system variability to be separated into repeatability and reproducibility, and part variation.

Repeatability = $\frac{5.15\overline{\overline{R}}}{d_2}$ where $\overline{\overline{R}}$ is the average of the ranges for all appraisers and parts, and d_2 is found in appendix A with Z= the number of parts times the number of appraisers, and W= the number of trials.

Reproducibility = $\left(\frac{5.15\bar{x}_{range}}{d_2} - \frac{repeatability^2}{nr}\right)^{1/2}$ where $\bar{x}_{range}$ is the average of the difference in the average measurements between the appraiser with the highest average measurements, and the appraisers with the lowest average measurements, for all appraisers and parts, d_2 is found in appendix A with Z=1 and W= the number of appraisers, n is the number of parts, and r is the number of trials.

R&R = $\sqrt{repeatability^2 + reproducibility^2}$

The part variability is $V_p = \frac{5.15R_p}{d_2}$

3. ANOVA Method

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Statistic
Appraiser	SSA	a-1	$MSA = \frac{SSA}{a-1}$	$F = \frac{MSA}{MSE}$
Parts	SSB	b-1	$MSB = \frac{SSB}{b-1}$	$F = \frac{MSB}{MSE}$
Interaction	SSAB	(a-1)(b-1)	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F = \frac{MSAB}{MSE}$
Gage	SSE	ab(n-1)	$MSE = \frac{SSE}{ab(n-1)}$	
Total	TSS	N-1		

Table 1: Two way ANOVA

$$SSA = \sum_{i=1}^{a} \frac{(Y_{i..})^2}{bn} - \frac{Y^2_{...}}{N}$$

$$SSB = \sum_{j=1}^{b} \frac{(Y_{.j.})^2}{an} - \frac{Y^2_{...}}{N}$$

$$\text{SSAB} = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{(Y_{ij.})^2}{n} - \frac{Y_{..}^{2}}{N} - SSA - SSB$$

$$\text{TSS} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} Y_{ijk}^{2} - \frac{Y^{2}_{..}}{N}$$

SSE = TSS - SSA – SSB- SSAB

a = number of operators

b = number of parts

n = number of trials

N = total number of readings (abn)

Repeatability = $5.15\sqrt{MSE}$

Reproducibility = $5.15\sqrt{\dfrac{MSA - MSB}{bn}}$

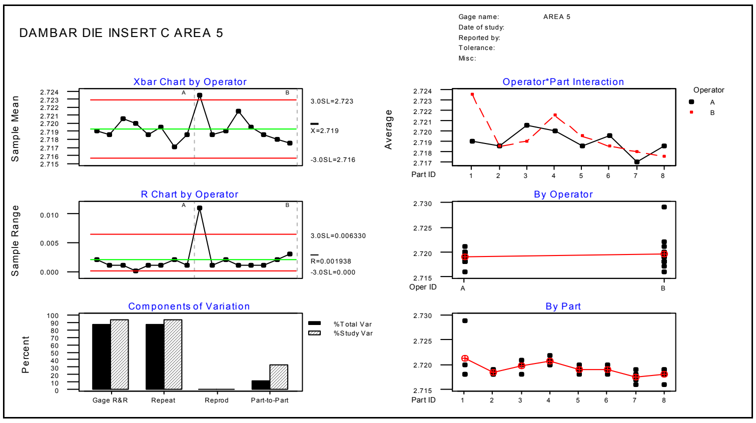
Operator and part interaction = I = $5.15\sqrt{\dfrac{MSAB - MSE}{n}}$

R&R = $\sqrt{repeatability^2 + reproducibility^2 + I^2}$

Measurement system part by part variation = $V_p = 5.15\sqrt{\dfrac{MSB - MSAB}{an}}$

Total measurement system variation = $V_T = \sqrt{R\&R^2 + V_p^{\,2}}$

4. Results



5. ANOVA Table With Operator*Part Interaction

Source	DF	SS	MS	F	P
Parts	7	4.72E-05	6.75E-06	1.80095	0.22785
Operators	1	2.53E-06	2.53E-06	0.67580	0.43814
Oper*Part	7	2.62E-05	3.75E-06	0.77327	0.61811
Repeatability	16	7.75E-05	4.84E-06		
Total	31	1.53E-04			

Gage R&R

Source	*VarComp*	*StdDev*	*5.15*Sigma*
Total Gage R&R	4.51E-06	2.12E-03	1.09E-02
Repeatability	4.51E-06	2.12E-03	1.09E-02
Reproducibility	0.00E+00	0.00E+00	0.00E+00
Operator	0.00E+00	0.00E+00	0.00E+00
Part-To-Part	5.59E-07	7.48E-04	3.85E-03
Total Variation	5.07E-06	2.25E-03	1.16E-02

Source	%Contribution	%Study Var
Total Gage R&R	88.97	94.32
K		
Reproducibility	0.00	0.00
Operator	0.00	0.00
Part-To-Part	11.03	33.21
Total Variation	100.00	100.00

Number of Distinct Categories = 0

The XBar Chart by operator shows that most of the points are inside the control limits, indicating the observed variation is mainly due to the measurement system. The percent contribution from total Gage R&R is larger than that of Part-to-Part, so most of the variation is due to the measurement system primarily repeatability and little due to the differences between parts. The Response by operator chart shows a nearly level line, so there is not much differences between operators. however there are slight differences between parts, as shown by the response by part chart.

6. Discusion and conclusion

After analyzing the results using ANOVA and Xbar & R method, we came to a conclusion that the overall variation is caused by the measurement system that is being used. Most of the observed variation is due to repeatability. This means, that there is more variation when the measurement system uses one operator to measure the parts given, then when two operators are used. The response by operator also overall shows that there are not much differences between the measurements taken by the two operators. ANOVA method is a more accurate way to conduct this anlysis as it acounts for operator by part interaction.

7. References

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