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Climate Change: The Evidence from Central England Temperatures

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Abstract

The main aim of this paper is to investigate whether there is evidence to suggest that the underlying trend of the pattern of Central England Temperature (CET) series varies over time. In addition, it is also of particular interest in describing the structure of the data. For this paper, the data is a series of 10-year averages of the CET series from 1663 to 2002. There is no doubt that a linear regression trend in time with an MA (2) noise model describes the series adequately. The model suggests that there is evidence of an overall warming trend, and generally, the temperature is estimated to rise approximately 0.0252 °C over a decade. Moreover, the estimated 95% confidence interval for the slope is (0.01148, 0.03892).

Keywords: Central England Temperature; Box-Jenkins approach and regression models with ARMA errors.

1. Introduction

The Central England Temperature (CET) series is probably the best known and is one of the longest consistent records of temperature in existence for anywhere in the world. The CET is representative of a roughly triangular area of the United Kingdom enclosed by Bristol, Lancashire and London. There are many questions of interest, particularly in connection with climate change, including whether there are any regularities in temperature fluctuations, whether there is evidence of a consistent rise in temperature going beyond natural fluctuations, etcetera. The most disputable question in current climate change research is over attribution of recent climate change to either natural processes or human activities over the period of the instrumental record. However, in this paper, the principal aims are to describe the structure of the CET series, and to fit an appropriate time series model to the data in order to discuss evidence of climate change over time as experienced in Central England.

The construction of this temperature record, which extends back to 1659, was the lifetime work of Professor Gordon Manley. The series is now continually updated by the Hadley Centre for Climate Prediction and Research, which is part of the Meteorological Office in United Kingdom. The monthly mean surface air temperatures are expressed in degrees Celsius (°C) for the period from January 1659 up to present [4]. The data is discussed by Manley (1974) and Parker *et al.* (1992). Manley draws attention to the fact that the data values prior to 1723 are rather unreliable and are rounded to 0.5 °C. In addition, values since 1974 have been adjusted by 0.1 °C - 0.2 °C to allow for urban warming.

For this paper, the data is a series of 10-year averages of the CET series for the decades 1663-1672 to 1993-2002, which aims to allow a broad view of movements in temperature and a simpler analysis. The data set is arranged in two columns. The first column gives the first year of each decade and the second column corresponds to the average temperatures for each decade. The average temperatures for each decade are calculated from yearly data by totalling the corresponding annually values and dividing by 10. All analyses were performed using S-Plus version 7.0.

2. Preliminary Analyses

Time plot

As always in any preliminary stage of a time series analysis, producing a time plot to identify the structure represented by the sequence of observations and understand the variation in the series is crucial. The time plot of temperatures is shown in Figure 1. A quick visual inspection of the graph shows that the temperatures have a general upward trend over 34 decades, ignoring short-term fluctuations. In addition, there is arguably a sign of cyclic pattern with periods 40-50 years, though the amplitude changes apparently erratically. Besides, the plot also indicates perhaps two 'outliers', i.e. values that seem out of line with the others, in 1693-1702 (8.288 °C) and 1993-2002 (10.178 °C). On the other hand, there is no justification for discarding them as no evidence suggests that these observations were incorrectly recorded.

Further detailed scrutiny of the plot might reveal the CET record has some turning points. The horizontal line shows the average temperature in Central England for the period from 1663 to 2002. Undeniably the decadal values after the 20th century have been higher than before. The series has a lower mean level and possibly a downward trend in the early years, followed by a period of stability, though irregular oscillations are obvious, and ends with an apparent rising trend. It is natural to suppose that a linear or quadratic trend is latent in the structure, though the latter is rather unlikely.

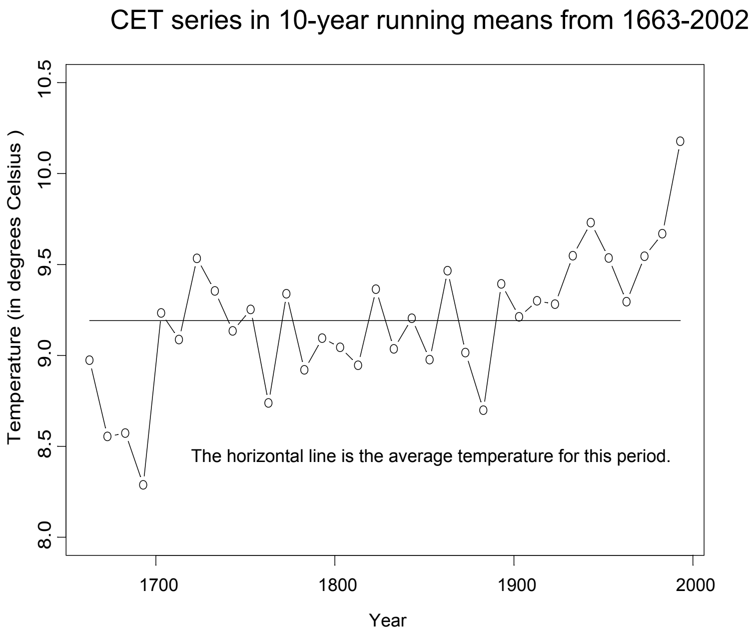

Figure 1. Time plot of the CET series.

Estimation of trend

It is important to note that for simplicity, it is assumed that the CET series is non-seasonal. Furthermore, seasonality is also not apparent in Figure 1. In the absence of seasonal component but only the presence of trend component and random noise component, fitting a straight line or low order polynomial using least squares may be a choice to estimate the trend in the series.

The results of fitting the simple linear and quadratic fits to the data are summarized in Table 1. Table 1 indicates that a linear warming trend but not a quadratic trend is hidden in the structure of the data. The estimated rate of rise in temperature per year is about 0.0024 °C.

Table 1. Estimated parameters for the linear and quadratic fits.

Coefficient	Linear fit			Quadratic fit		
	Value	Standard error	*p*-value	Value	Standard error	*p*-value
Intercept	4.7736	0.9259	< 0.001	26.5298	19.1657	0.1762
Linear term	0.0024	0.0005	< 0.001	-0.0215	0.0210	0.3151
Quadratic term	-			< 0.001	< 0.001	0.2645

3. Time Series Analyses

There are many classes of time series models in the literature, but for this paper, the focus is on using Box-Jenkins approach, i.e. fitting an autoregressive integrated moving average (ARIMA) model, to describe and quantify the CET series. Besides, investigations on regression models with ARMA errors were also carried out.

Box–Jenkins approach

Classical Box-Jenkins models assume that the time series is stationary. Loosely speaking, a time series $\{X_t, \ t = 0, \ \pm 1, ...\}$ is said to be stationary if it has statistical properties (e.g., the mean and the variance) similar to those of the "time-shifted" series $\{X_{t+h}, \ t = 0, \ \pm 1, ...\}$, for each integer h (Brockwell and Davis, 2002).

Stationarity can be examined from a time plot. It can also be detected from the autocorrelation (ACF) plot. Specifically, an ACF plot with very slow decay suggests non-stationarity. In other words, a time series is stationary if the ACF has very few significant spikes at very small lags and then cuts off drastically or dies down very quickly.

To ensure stationarity, Box and Jenkins recommend the differencing approach so that the differenced observations resemble a realization of stationary time series. Box and Jenkins (1976) developed a methodology for fitting ARMA models to differenced data. These are known as ARIMA models. The Box-Jenkins methodology consists of a four-step iterative procedure, i.e. tentative identification, estimation, diagnostic checking and forecasting. However, since forecasting is not of any concern in this paper, only the first three steps will be iteratively performed to search for a satisfactory model.

Model identification

Visual inspection of the time plot suggests that trend component might present in the series. Also, according to the above preliminary analysis, the linear term is significant, suggesting the CET series is non-stationary. On the other hand, examination of the correlogram of the CET record does not give indication of lack of stationarity as the sample autocorrelations decay to zero after lag 2. In other words, there are two obvious spikes (except at lag zero), and the rest are essentially zero. Moreover, the plot of sample ACF shows a slight amount of positive autocorrelation. Thus, there are reservations whether to difference the original series. In other words, it is not entirely obvious whether the CET series is stationary or not. Perhaps the series is short, consists of only 34 observations, and does not have a reasonable stable form of non-stationarity, so there is no definitive or obvious pattern of non-stationary emerges in the correlogram.

It has to be recognised that differencing has its limitations, working only for considerably stable forms of non-stationarity. Differencing tends to introduce negative correlation. If the series initially shows strong positive autocorrelation, then a non-seasonal difference will reduce the autocorrelation and perhaps even drive the lag 1 autocorrelation to a negative value. If the lag 1 autocorrelation is zero or even negative, then the series does not need further differencing. If the lag 1 autocorrelation is -0.5 or more negative, the series may be overdifferenced [3].

Assuming that the CET series is non-stationary, it appears as though one differencing operation is sufficient to transform the series into stationarity with no apparent trend. But, there is a pattern of changes of sign from one observation to the next in the time series plot of the differenced data, possibly suggesting the signs of overdifferencing. Furthermore, the sample ACF of the differenced series confirms that the series is overdifferenced as there is a negative spike at lag 1 that is close to 0.5 in magnitude. However, differencing that yields a slightly overdifferenced series is expected as the CET series shows a slight amount of positive autocorrelation.

If the ACF of the differenced series displays a sharp cutoff and/or the lag 1 autocorrelation is negative – i.e. if the series appears slightly "overdifferenced" – then consider adding an MA term to the model [3].

The sample ACF of the differenced series suggests that a moving average process of order 1 is an appropriate model for the lag 1 differences. Thus, an initial attempt would be to fit an ARIMA (0,1,1) for the original data (or ARMA (0,1) for the differenced series). Conversely, the sample partial autocorrelation plot of the differenced data indicates the possible suitability of an AR (1) model for the differences as the sample partial autocorrelation cuts off beyond lag 1.

The extensive and major tools used in this identification phrase, i.e. correlograms of the autocorrelation and partial autocorrelation functions, do not provide any definitive answer but throw some light on some possible number of parameters to be estimated.

Model fitting [1], selection and validation

Several plausible models were fitted and compared in order to obtain a 'best' model that reflects the structure of the series. First, two models, i.e. ARIMA (1,1,0) and ARIMA (0,1,1) with a constant term, were fitted based on the indications as to order provided by the plots of sample autocorrelation and sample partial autocorrelation of the differenced data. The results are summarized in Table 2. On comparing the Akaike Information Criterion (AIC), residual mean squares and the log-likelihood of the fitted models, the preferable model obviously is the ARIMA (0,1,1) as it has smaller AIC and residual mean squares values but larger log-likelihood value.

Table 2. Fitted ARIMA (1,1,0) and ARIMA (0,1,1) models.

Model: ARIMA (1,1,0)	Value	Model: ARIMA (0,1,1)	Value
AR (1) (standard error)	-0.5162 (0.1541)	MA (1) (standard error)	-0.7790 (0.1281)
Constant (standard error)	0.0363 (0.0358)	Constant (standard error)	0.0295 (0.0146)
AIC	22.43	AIC	20.32
Log-likelihood	-8.22	Log-likelihood	-7.16
Residual mean squares	0.0954	Residual mean squares	0.0878

Basic diagnostics from the fitted models, i.e. using plot of standardized residuals, autocorrelation function of the residuals and the *p*-values for Ljung-Box statistic, give no grounds to question the adequacy of the models. Overall, both are adequate models. However, the ARIMA (0,1,1) is a little better than the ARIMA (1,1,0).

Overfitting was carried out to check whether adding extra parameters in the MA model is worthwhile. The overfitted models, i.e. ARIMA (0,1,2) and ARIMA (1,1,1) with a constant term, have a larger AIC value and there is no abrupt change in the residual mean squares and log-likelihood. Furthermore, overfitting produces an estimate of MA (2) for ARIMA (0,1,2) that is easily consistent with the parameter being zero. On the other hand, for ARIMA (1,1,1), the estimated AR (1) is 0.1982 and its standard error is 0.1941. Hence, ARIMA (0,1,1) is still preferable in terms of parsimony. Parsimonious means choosing the model that has the fewest parameters and greatest number of degrees of freedom among all models that fit the data.

There is indication from the correlogram in the model identification stage that the original series might be in a form of stationarity, which means that the series should not be differenced at all before fitting any model. As there is reservation about non-stationarity of the original series, further investigations were carried out to search for ARMA models that may fit the original series. The plots of sample autocorrelation and sample partial autocorrelation of the original data suggest that some possible models for the original series are ARMA (1,0) and ARMA (0,2). Thus, both models were fitted with a constant term. By checking these models,

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[1] The model fitting in S-Plus was done by using the MASS library function `arima` which fits an ARIMA model of specified order by maximum likelihood, using a full likelihood based on the assumption of normality.

ARIMA (0,1,1) is still the 'best' model since their residual mean squares and AIC are larger and the log-likelihood is smaller compared to ARIMA (0,1,1). Moreover, diagnostics from the fit of ARMA (1,0) to the original series show evidence against this model as some of the *p*-values of the Portmanteau test are small. Thus, assuming the CET series is non-stationary and fitting the ARIMA model is a sensible option.

Regression models with ARMA errors

Time series models can include a regression-type dependence on one or more covariates. The regression variables may simply be a constant (intercept) term, a deterministic function of time, dummy variables to model outliers, or lagged values of another time series. The `arima` function can use the optional argument `xreg` to specify covariates. If a `xreg` term is included, a linear regression (with a constant term if `include.mean` is true) is fitted with an ARMA model for the error term.

It is clearly seen in Figure 1 that a trend may be present in the data. Moreover, in Section 2.2., it has been demonstrated that a linear trend can be fitted to the data by least squares, suggesting that the CET series is non-stationary. However, a straight line would be too simple to represent the average-over-a-few-decades behaviour of the temperature data. Accordingly, another possibility to modelling the data would be to fit regression models with ARMA errors using the `xreg` argument of `arima`.

To begin with the model building process, it is reasonable to consider fitting a linear time trend to the data with low order ARMA errors, e.g., a simple linear regression model with an AR (1) noise and a simple linear regression model with an MA (1) residuals. Table 3 summarizes the AIC, log-likelihood and residual mean squares values for nine fitted models.

Table 3. AIC, log-likelihood and residual mean squares values for various ARMA models for the residuals.

Model for the residuals	AIC	Log-likelihood	Residual mean squares
(1,0)	17.34	-4.67	0.07700
(0,1)	17.56	-4.78	0.07753
(1,1)	19.09	-4.55	0.07642
(1,2)	17.94	-2.97	0.06873
(2,1)	19.95	-3.97	0.07370
(2,0)	18.40	-4.20	0.07477
(0,2)	17.27	-3.63	0.07197
(3,0)	19.07	-3.54	0.07163
(0,3)	17.47	-2.74	0.06779

By checking the parameter estimates for all models and taking into account the statistics shown in Table 3, ARMA (1,2), ARMA (0,2) and ARMA (0,3) models for the residuals are equally good. However, it is found that ARMA (0,2) or MA (2) model for the residuals has the smallest value of the AIC statistic. Furthermore, MA (2) is more parsimonious than the other two. Also, there are no problems arise in the diagnostics check for this model. Hence, there is reason in favour of MA (2) model for the residuals.

Lastly, a quadratic trend plus MA (2) for the noise model, which is intended to investigate whether the hidden structure in the CET series is quadratic, was fitted. The results suggest that the underlying trend is probably a linear time trend as the estimated linear term is not significantly different from zero.

4. Results

In general, differencing and Box-Jenkins approach shows that one moving average parameter is necessary and sufficient to yield an effective but still parsimonious model of the process. The fitted model for the process is as follows:

$$X_t = X_{t-1} + \varepsilon_t - 0.779\varepsilon_{t-1} + 0.0295 \quad (1)$$

where X_t and ε_t are the observation and innovation at time *t* respectively. The estimated variance of ε_t is 0.0878. The constant in the ARIMA (0,1,1) model represents the mean of the differenced series, as there are no autoregressive parameters in the model, and therefore the linear trend slope of the original series. In other words, the constant represents the rate of rise in temperature per decade. The estimated coefficient for time effect is approximately 0.0295, implying that the rate of rise in temperature per decade is about 0.0295 °C or equivalently 0.00295 °C per year.

On the other hand, the regression method suggests the following model:

$$X_t = 8.7636 + 0.0252 \times t + a_t$$

$$a_t = \varepsilon_t + 0.2596\varepsilon_{t-1} + 0.3154\varepsilon_{t-2}, \quad \{\varepsilon_t\} \sim \mathrm{WN}(0, \sigma^2) \qquad (2)$$

$$\hat{\sigma}^2 = 0.07197$$

where X_t is the observation at time *t*, ε_t is the innovation at time *t* and $\hat{\sigma}^2$ is the estimate of σ^2. Model (2) suggests that the rise in temperature per decade is roughly 0.0252 °C. In other words, model (2) supports a possible warming trend over years as the linear term or slope is significantly different from zero.

To describe the structure of the data adequately and understand the underlying process, model (2) is recommended, though not definitive, since it has a lower AIC value than model (1). However, both models do give similar results concerned with the evidence of climate change. The estimated 95% confidence bounds for the slope are $0.0252 \pm 1.96(0.007)$, demonstrating that a significant increasing trend in the temperature of Central England during the years 1663-2002.

5. Conclusions

This paper is mainly concerned with the Central England Temperature record. The original aims, to describe the structure and understand the variation in the data, and to discuss evidence of change in the CET series over time on the basis of a simple but concise model, were successfully achieved.

Graphical exploration shows that the Central England has experienced roughly 3 different periods, i.e. cooling, stability and warming from 1663 up to 2002. Assuming seasonality is absent in the series, an ARIMA (0,1,1) model with a constant term was fitted and there is evidence that this model describe the data satisfactorily. Nevertheless, there are reservations about the ARIMA (0,1,1) model with a constant term as Chatfield (1996) recommends that there should be at least 50 observations in the input data with the aim of fitting any ARIMA models. Furthermore, several regression models with ARMA errors were fitted and the results suggest that a linear time trend plus MA (2) for the noise model is a better fit to the data. Thus, this model is chosen to be the final model to describe the CET series.

As the topic of interest is climate change, the aim is concentrating on estimation of the trend component. There is reason to believe that the deterministic trend takes a linear form. Conclusion relating whether there is an overall warming trend is drawn based on model (2). The model suggests that the data exhibits evidence of temporal dependence. In other words, there is evidence of an overall warming trend, and in general, there is only slight increase in the temperature over a decade, around 0.0252 °C. The estimated 95% confidence interval for the slope is (0.01148, 0.03892). In short, climate change is detected and there is evidence of an increase in the temperature in the CET series.

6. References

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