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Pattern of Exchange Rate Distributions

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Abstract

This analysis is conducted within the contect of a stochastic version of the Dornbusch (1976) overshooting model, following the Miller and Weller (1991) extension of Krugman (1991), with shocks to aggregate supply. Hence, three realistic features are added to the Krugman (1991) model: (i) home and foreign products are imperfect substitutes in consumption so that purchasing power parity is relaxed, (ii) prices and wages are sluggish, and (iii) there are intra marginal as well as infra-marginal interventions in the foreign exchange market. The objective of this paper is thus to show that mean reversion introduced by sluggish adjustment of prices and wages combined with a degree of intra-marginal intervention can result in a hump-shaped exchange rate distribution and thereby explain the empirical evidence. We would like to show that the hump shape is more likely to occur when the degree of monetary accommodation is close to what is needed to peg the nominal exchange rate (so that the fundamental has to move far away from its mean for the exchange rate to reach its boundaries).

1. Introduction

Most of the existing work on nominal exchange rate bands is based on Krugman (1991), and adopts the unrealistic assumptions of purchasing power parity and fully flexible wages and prices, and thus assumes full employment. Notable exceptions are Miller and Weller (1991), who extend Krugman (1991) to a stochastic version of the familiar exchange rate overshooting model of Dornbusch (1976), thus allowing for sluggish price movements and imperfect substitution between home and foreign goods in consumption. Our model of a small open economy extends Miller and Weller (1991) to allow for different types of nominal exchange rate regimes, each one of them characterized by a particular feedback rule for the money supply. We extend Miller and Weller (1991) by allowing for intra-marginal accommodation and intervention in the foreign exchange market. Our model may be summarized by the following equations:

$$y = -\eta(i-\pi) + \delta(e - p^* - p),\ \eta > 0, \quad 0 < \delta < 1 \tag{1}$$

$$m - p = y - \lambda i,\ \lambda > 0 \tag{2}$$

$$dp = \phi(y - y^F)dt + \pi dt + \sigma dz,\ dz \sim IN(0,dt), \quad \phi > 0 \tag{3}$$

$$E_t(de) = (i - i^*)dt \tag{4}$$

$$m = m(p) \tag{5}$$

where m, y, y^F, p, p^*, q, e and dz denote logarithms of the nominal money supply, the level of aggregate demand, the full-employment level of output, the home price level, the foreign price level, the consumer price index (CPI), the nominal exchange rate (price of one unit of foreign currency in terms of domestic currency units) and a supply shock, respectively, i and i^* denote the home and foreign nominal interest rate, respectively, and π denote the rate of core inflation.

The home country specializes in the production of a good with price p and the foreign country specializes in the production of good with price p^*. There is imperfect substitution between home and foreign goods. Equation (1) is the IS-curve and shows that aggregate demand increases when the real interest rate declines or the real exchange rate depreciates. The real interest rate is simply the nominal interest rate minus the core inflation rate. Equation (2) is the LM-curve and says that the velocity of circulation increases with the nominal interest rate. Equation (3) is the Phillips-curve which shows that inflation in wages and producer prices occurs when there is excessive demand for goods, and that deflation occurs when there is unemployment. The speed at which the labor market clears, i.e the degree of price and wage flexibility, corresponds to the parameter ϕ. For convenience, the full-employment level of output is assumed to equal λi^* (i.e. $y^F = \lambda i^*$). This guarantees that in steady state, i.e. when the expected change in the price level and the exchange rate is zero, nominal money supply and the price level are both equal to μ. It also ensures that, if there is full monetary accommodation, i.e. a PPP exchange rate rule, the real exchange rate is pegged, other real variables are constant as well, and expected price and nominal interest rate changes are zero. Supply shocks (z) follow a Brownian motion and correspond to positive shocks to nominal wages; z follows an independent Wiener process with zero mean and instantaneous variance equal to σ. Producer prices are a constant mark-up on unit labor costs, so that supply shocks may be interpreted as negative shocks to labor productivity. The money supply is stable in steady state, so core inflation (π) being the trend rate of inflation is zero. The advantage of this simple specification is that one can unambiguously determine that the system is saddle-path stable. If aggregate demand depends on the real consumption interest rate, i.e. the nominal interest rate minus the rational expected rate of change in the CPI, there is a possibility of an unstable spiral (i.e. higher inflation depresses the real interest rate, boosts aggregate demand and thus induces even higher inflation). Our definition of core inflation in the definition of the real interest rate avoids these indeterminacies and simplifies the algebra. Equation (4) is the uncovered interest parity condition. Risk-neutral arbitrage ensures that an interest differential in favour of the domestic country can only be sustained if the currency is expected to depreciate in the future, i.e. if the currency is currently over-valued. Finally, equation (5) is the feedback rule for money supply in response to price changes. All exchange rate regimes considered in the sequel are characterized by a special case of this feedback rule for monetary policy.

2. Unrestricted Dirty Float: The Case of No Band on The Exchange Rate

To facilitate comparison with a regime in which an exchange rate band is present, we first consider the case of an unrestricted dirty float. Hence, there is no exchange rate band and the money supply rule is linear:

$$m = \mu + \beta(p - \mu) = (1-\beta)\mu + \beta p\,,\ \beta < 1. \tag{6}$$

This feedback rule says that, when prices exceed their long-run value μ (i.e. the exogeneous component of the money supply), the monetary authorities accommodate and raise the money supply. The degree of monetary accommodation is given by the coefficient β. The advantage of such a simple rule is that it is easily understood by the market.

For simplicity, and without any loss of generality, we normalize all foreign variables to zero. Using (6), the reduced form equations of the model under a dirty float are given by:

$$dp = \phi(\eta + \lambda)^{-1}\{-[\delta\lambda + \eta(1-\beta)]p + \delta\lambda e + \eta(1-\beta)\mu\}dt + \sigma dz \tag{7}$$

$$Ede = (\eta + \lambda)^{-1}[(1-\beta-\delta)p + \delta e - (1-\beta)\mu]dt \tag{8}$$

The price level is a predetermined, backward-looking variable, while the exchange rate is a non predetermined, forward-looking variable which jumps if private agents suddenly anticipate a change in future policy. The rational expectations equilibrium must therefore be a stable saddle-path solution. This requires one eigenvalue with a negative real part and one eigenvalue with a positive real part, which is ensured as long as $\beta < 1$ holds. To find the unique, non-explosive rational expectations solution, we postulate a linear saddle-path:

$$e - mean(e) = \omega[p - mean(p)] \tag{9}$$

where $mean(p) = mean(e) = \mu$. Upon substitution of (9) into (7) and (8) and equating coefficients on p, we find:

$$\omega = \left(\frac{\phi\eta(1-\beta) + \delta(1+\lambda\phi) - \sqrt{[\phi\eta(1-\beta) + \delta(1+\lambda\phi)]^2 + 4\lambda\phi\delta(1-\beta-\delta)}}{2\lambda\phi\delta} \right) \tag{10}$$

In fact, there is another solution for ω as well. However, that solution can be ruled out because it does not satisfy the requirement that the adjustment of prices along the saddle-path is a stable process.

The slope ω of the saddle-path increases if the coefficient of monetary accommodation (β) increases. If $\beta = 1-\delta$, then $\omega = 0$. If $\beta = 1$ and $\phi\lambda \le 1$, then $\omega = 1$. If $\beta = 0$ and $\delta \ge \frac{1}{2}$, $\omega > -1$. For low degrees of monetary accommodation, i.e. $\beta < 1-\delta$, there is a negative correlation between the nominal exchange rate and the price level, i.e. the saddle-path slopes downwards. This implies that the nominal exchange rate overshoots in response to an unanticipated and permanent change in the long-run money supply (cf., Dornbusch, 1976). For high degrees of monetary accommodation, $1-\delta < \beta < 1$, the saddle-path slopes upwards, which implies undershooting of the nominal exchange rate. The turning point between overshooting and undershooting of the nominal exchange rate in response to an unanticipated permanent change in the money supply corresponds to the coefficient of monetary accommodation that ensures a fixed nominal exchange rate (i.e. $\beta = 1-\delta$), so that $\omega = 0$. This nominal exchange rate regime will be referred to as a peg.

It is easy to see that for $\beta < 1-\delta$, ω is increasing in ϕ, while for $1-\delta < \beta < 1$, ω is decreasing in ϕ. Therefore, as ϕ increases, the relationship between ω and β pivots around the peg ($\beta = 1-\delta$) towards the horizontal axis. In the extreme case of a classical model with full employment (i.e. $\phi \to \infty$), $\omega \to \infty$ and all transitional dynamics disappear.

Under a peg the monetary authorities use un-sterilized interventions to fix the exchange rate. If the foreign interest rate exceeds the domestic interest rate, there is an incipient capital outflow and pressure for the currency to depreciate. Under a peg, the central bank defends the exchange rate by selling foreign reserves and buying its own currency. As a result, the money supply falls until the domestic interest rate is pushed up to the level of the foreign interest rate. Hence, a peg corresponding to $e = e_p$ implies that the domestic interest rate is anchored to the foreign interest rate, and that an independent domestic monetary policy is infeasible:

$$m = (1-\delta)p + \delta e_p \equiv \beta(p-\mu) + \mu \tag{11}$$

Hence, a peg at $e = e_p$ corresponds to a very specific (linear) money supply rule, namely an accommodation coefficient of $\beta = 1-\delta$ and a long run component of the money supply of $\mu = e_p$. Upon substitution of these results in (7), we find that prices follow an Ornstein-Uhlenbeck process (Karlin and Taylor, 1981, p.172):

$$dp = -\phi\delta(p - e_p)dt + \sigma dz \tag{12}$$

which is a mean-reverting process. Clearly, the asymptotic mean of this process is mean $(p) = \mu = e_p$. The expected speed of mean reversion towards the steady state increases while the variances of output and prices (i.e. $\text{var}(y) = \delta^2 \text{var}(p) = \frac{\frac{1}{2}\sigma^2\delta}{\phi}$) decrease with the degree of labor market flexibility (ϕ). In a classical world in which markets clear instanteneously the variances of output and prices are zero. If aggregate demand is more sensitive to relative prices (higher δ), the variance of output is higher but the variance of the price level is lower. The variance of inflation over the unit interval (say, Δp) is given by $\text{var}(\Delta p) = 2[1-\exp(-\phi\delta)]\text{var}(p)$. Full employment ($\phi \to \infty$) implies that both the variance of the price level and of the inflation rate are zero.

3. Dirty Float With A Nominal Exchange Rate Band

As already mentioned, one of the crucial elements in the explanation of the observed hump-shaped distributions of exchange rates within a nominal exchange rate band is the presence of intra-marginal interventions in the foreign exchange market. Hence, below we introduce a band on the exchange rate and to capture the presence of intra-marginal interventions we allow for a non-zero coefficient of monetary accommodation (β), cf. the rule (6), if the exchange rate is inside the band. We assume, for ease of notation, that the long run component of the money supply is zero (i.e. $\mu = 0$). In order to solve for the exchange rate solution within its band, we postulate instead of the linear saddle-path solution (9) a twice differentiable function for the solution:

$$e = \Omega(p) \tag{9a}$$

Use of Ito's Lemma, $d\Omega = \Omega' dp + \frac{1}{2}\sigma^2 \Omega''$ dt, yields a second-order nonlinear differential equation:

$$\frac{1}{2}\sigma^2\Omega''(p) + \phi(\eta+\lambda)^{-1}\{-[\delta\lambda + \eta(1-\beta)]p + \delta\lambda\Omega(p)\}\Omega'(p) - (\eta+\lambda)^{-1}[(1-\delta-\beta)p + \delta\Omega(p)] = 0. \tag{13}$$

This equation yields a time-invariant relationship between the nominal exchange rate and the price level. Note that there are only two linear solutions, which correspond exactly to the stable and the unstable arm of the saddle-path solution of the unrestricted dirty float . These linear solutions are, of course, not compatible with the presence of finitely wide nominal exchange rate bands.

To solve for the exchange rate when it is inside the band, it is necessary to specify the nature of the interventions undertaken by the monetary authorities when the exchange rate reaches the boundaries of its band. The band on the nominal exchange rate consists of an upper bound (e^U) and a lower bound (e_L). The authorities ensure that the exchange rate does not move outside this band by imposing thresholds $p^U \equiv \Omega^{-1}(e^U)$ and $p_L \equiv \Omega^{-1}(e_L)$ on the price level (the "fundamental"). We focus only on exchange rate solutions symmetric about the origin by normalizing the central parity to zero and assuming that $p^U = -p_L$ (so that $e^U = -e_L > 0$). If the price is between p^U and p_L, the exchange rate is inside its band and the degree of monetary accommodation or intra-marginal intervention to price changes in β, while, beyond these thresholds, the degree of accommodation switches to the one (i.e. $1-\delta$) that is needed to keep the exchange rate fixed at the upper, respectively, lower boundary of its band. The complete description of the money supply rule which supports this policy is given by the following piecewise-linear monetary accommodation rule (this piecewise-linear specification does not allow intra-marginal interventions to depend on how close the exchange rate is to the edges of the band. Allowing for this complicates the solution without changing the qualitative features of the analysis much):

$$\begin{aligned}
m &= \beta p, \quad \textit{for} \quad p^U < p < p_L \quad \textit{if} \quad 0 \le \beta < 1-\delta \quad \textit{and} \quad p_L < p < p^U \quad \textit{if} \quad 1-\delta < \beta < 1, \\
m &= (1-\delta)p + \delta e_L, \quad \textit{for} \quad p \ge p_L \quad \textit{if} \quad 0 \le \beta < 1-\delta \quad \textit{and} \quad p \le p_L \quad \textit{if} \quad 1-\delta < \beta < 1, \\
m &= (1-\delta)p + \delta e^U, \quad \textit{for} \quad p \le p^U \quad \textit{if} \quad 0 \le \beta < 1-\delta \quad \textit{and} \quad p \ge p^U \quad \textit{if} \quad 1-\delta < \beta < 1,
\end{aligned} \tag{14}$$

Note that (14) makes use of $\Omega' < 0$ for $0 \le \beta < 1-\delta$, and $\Omega' > 0$ for $1-\delta < \beta < 1$, which correspond to an inverted and regular S-shaped solution of $\Omega(.)$, respectively. The switch points $p^U \equiv \Omega^{-1}(e^U)$ and $p_L \equiv \Omega^{-1}(e_L)$ follow implicitly from the smooth pasting conditions, which are the appropriate boundary conditions. In Miller and Weller (1991) the band is sustained by infinitesimal adjustments of the money supply if the exchange rate reaches the boundaries of its band. However, independent of the amount of cumulated intervention, the exchange rate always returns inside its band if the price movement reverses after intervention. This intervention policy implies a non-unique relationship between the price level and the exchange rate, namely one that depends on the current level of money supply. The intuition is that changes in the money supply, as a result of interventions, temporarily change the long-run equilibrium consistent with the current level of money supply. Hence, the exchange rate solution in the band shifts and becomes relatively steep in the middle of the band. Combined with the global mean-reversion in the price level, this implies a concentration of probability mass near the edges of the band. Although it is not possible to obtain a closed-form expression for all solutions to differential equation (13), one can characterize the solutions qualitatively. It follows that there is a unique solution in the band which fulfills the smooth pasting conditions (for $1-\delta < \beta < 1$ this is guaranteed if $\phi\lambda \leq 1$). For $0 \leq \beta < 1-\delta$ the solution is downward sloping and for $1-\delta < \beta < 1$ it is upward sloping. It is strictly concave in the upper half of the band, and strictly convex in the lower half of the band. If the price level crosses p^U (or p_L) from the interior of its band, the nominal interest rate differential, i.e. i-i*, jumps from a negative (positive) value to zero. Hence, the monetary authorities have to implement a discrete contraction (expansion) of the nominal money supply in order to raise (depress) the interest rate and prevent the exchange rate from moving outside its band. Because the market anticipates this regime switch at the boundaries of the band, the exchange rate solution bends and becomes horizontal as the economy approaches the boundaries. This is what is known as the "honeymoon effect".

4. Monte-Carlo Simulation

Because we do not have analytical closed-form solutions for the unconditional exchange rate distributions when there is a band on the exchange rate, we can use Monte Carlo simulations to reproduce the hump-shaped pattern of the exchange rate distributions. The choice of parameter values is determined by the considerations outlined above. Given the variance (σ^2), which determines the overall variability of the system, one needs an accommodation coefficient (β) which is close enough to $1-\delta$, and at the same time a degree of labor market flexibility (ϕ) which is not too small. Various experiments suggest that there is a wide range of parameter values consistent with a hump-shaped pattern of the exchange rate distribution. Moreover, we conjecture that for a given degree of labor market flexibility ($\phi > 0$), it is always possible to choose β so close to $1-\delta$, that the implied bandwidth for the price level (i.e. the gap between p_L and p^U) becomes so large that the percentage of time that the exchange rate is at its boundaries is negligible.

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