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# Theoretical Welfare Cost Analysis to Reduce Carbon Dioxide Emissions

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## ABSTRACT

This paper focuses on insurance against the small probability of causing really catastrophic climate change may justify significantly curbing CO2 emissions. Such extreme non-linearities maybe exist. However predicting future global climate changes is extremely hazardous, and no-one can rule out the possibility of surprise. This paper uses indirect method to get the possible scenarios that could occur, nor what costs or subjective probabilities to attach to most of the catastrophes that have been suggested.

***Keywords:*** *welfare cost, carbon dioxide emissions, model without adjustment costs, model with adjustment Cost.*

## 1. Introduction

Economic analysis has so far tended to warn against radical measures to curb current emissions of CO2, which are thought by many scientists to be enhancing the effect, and thereby creating the possibility of future global warming and other changes in climate.

The case for taking some action now requires some source of convexity in the instantaneous costs of reducing emissions – if costs were linear, there is no economic gain from smoothing the adjustment over time which arises, at least in theory, because of:

a. concave utility,
b. concavity of production, which implies that the marginal cost of reducing an input (such as fossil fuels) is increasing.
c. Adjustment costs due to some inputs being very costly to shift to less $CO_2$ intensive production activities in the short run.

This paper is concerned with estimating whether curbing CO2's now really has much effect on welfare losses in future worst case scenarios when atmospheric concentrations have to be stabilized very quickly.

## 2. Model

Defining the value function, accumulation of CO2 in the atmosphere and steady state emissions by:

$$V\left(\hat{M}_0, E_0, \hat{M}^*\right) \qquad \hat{M}^* > \hat{M}_0 \tag{1}$$

which is the maximized value of utility, for the world economy, given that atmospheric concentrations must be stabilized at $\hat{M}^*$, and an inherited concentration level of $\hat{M}_0$ and $CO_2$ emissions rate $E_0$. Clearly $\frac{dV}{d\hat{M}^*} > 0$, since the target is less restrictive, and $\frac{dV}{d\hat{M}^*} < 0$, as there is smaller amount of the atmosphere which can be safely filled with $CO_2$ emissions.

Finally $\frac{dV}{dE_0} < 0$ to the extent that there is short run adjustment costs incurred in changing emissions levels.

Suppose utility is $V(\infty)$ when $\hat{M}* = \infty$. This paper is concerned with estimating:

$$\frac{V(\infty) - V(\hat{M}_0, E_0, \hat{M}*)}{V(\infty)} \quad (2)$$

that is, the percentage reduction in utility when a stabilization target is suddenly adopted.

The accumulation of $CO_2$ in the atmosphere is assumed where:

$$\dot{M}(t) - \gamma E(t) - \delta M(t) \qquad 0 < \gamma, \quad \delta < 1 \quad (3)$$

*M*(*t*) is the atmospheric concentration of $CO_2$ above the pre-industrial level (that is, due to human activities), and *E*(*t*) is instantaneous global emissions of $CO_2$.

From (3), the steady state emissions level *E**, given a ceiling for atmospheric concentrations of *M** is

$$E* = \frac{\delta M*}{\gamma} \quad (4)$$

## 2.1. The Model Without Adjustment Costs

We assumes there are no short run adjustment costs to changing emissions levels (therefore $E_0$ is not in the value function), should a stabilization target be adopted, given concave utility and production.

The instantaneous resource constraint (3) can be converted to a present value constraint, which will sometimes be the more convenient to use below. Adding and subtracting $\gamma E*$ to the right hand side of (3), and using (4), gives

$$\dot{M}(t) = \gamma\left[E(t) - E*\right] + \delta\left(M* - M(t)\right) \quad (5)$$

And defining $\Delta M(t) = M* - M(t)$, the amount of $CO_2$ free atmosphere at time *t*, then:

$$\Delta\dot{M}(t) = -\gamma\left[E(t) - E*\right] - \delta\Delta M(t) \quad (6)$$

Therefore when emissions are held at *E**, the stock of $CO_2$ free atmosphere depreciates at rate $\delta$. Integrating forward and setting $t = 0$ gives the initial present value constraint.

$$\Delta M_0 = \gamma\int_0^\infty e^{\delta t}\left[E(t) - E*\right]dt \quad (7a)$$

$$\Delta M_0 = M* - M_0 \quad (7b)$$

This just says that the sum of consumption of $CO_2$ free atmosphere over time equals the initial amount $\Delta M_0$. Deferring emissions till the future has a negative rate of return, since emissions depreciate at rate *δ*.

The present value of utility from future income, is given by:

$$\int_0^\infty e^{-\rho t} u\left(y\left(t\right)\right) dt \tag{8}$$

where $\rho > 0$ is the pure rate of time preference, and *y*(*t*) is income at time *t*.

Gross world income is assumed to be growing exogenously at rate *x* (there is no consumption/saving or labor/leisure choice in the model), thus:

$$y(t) = e^{xt}\bar{y} - \Delta y(t) \tag{9}$$

where $\bar{y}$ is the year observed, and $\Delta y\left(t\right)$ is the cost of holding emissions below non-abatement levels at time *t*.

Instantaneous utility is assumed to take the standard CES (Constant Elasticity Substitutions) form, thus:

$$\begin{aligned} u\left(y\left(t\right)\right) &= \frac{\left(y\left(t\right)\right)^{1-\sigma}}{1-\sigma} \text{ for } \sigma > 0 \text{ and } \neq 1 \\ u\left(y\left(t\right)\right) &= \ln\left(y\left(t\right)\right) \text{ when } \sigma = 1 \end{aligned} \tag{10}$$

where $\sigma$ is the elasticity of marginal utility with respect to income.

The cost of emissions abatement is given by:

$$\Delta y\left(t\right) = G\left[\bar{E}e^{at} - E\left(t\right), t\right] \tag{11}$$

where $\bar{E}$ is the observed year emissions levels, hence $\bar{E}e^{at}$ is what emissions would be in the absence of abatement policies at time *t*. *E*(*t*) is the actual level of emissions at time *t*. Therefore $\bar{E}e^{at} - E\left(t\right) \geq 0$ is the instantaneous amount of emissions reduction. *G* is a convex function, since $CO_2$ emissions, which are primarily caused by burning fossil fuels, are basically an input, and production functions are typically concave with respect to inputs.

## 2.2. The Model with Adjustment Cost in Changing the Composition of the Capital stock

In practice, reducing CO2 emissions in the most cost effective manner over the long run will involve some change in the composition of the economy's capital stock. That is, capital needs to be shifted into production activities which are relatively less CO2 –intensive. To the extent that capital is not costlessly variable over the short run, additional adjustment costs will be incurred, which are an increasing function of the speed of adjustment. In a future scenario when a stabilization target for atmospheric concentrations is adopted, the present value of these adjustment costs will be lower if action to reduce CO2 emissions had previously been taken, since (i) a less CO2 – intensive capital stock will be inherited and (ii) the stock of CO2 free atmosphere is greater, allowing for a more gradual adjustment.

It is assumed that the composition of the capital stock which can be thought of as the ratio of capital in $CO_2$ emitting to that in non $CO_2$ emitting sectors can be adjusted at the end of each year, and remains fixed at that level till the end of next year. $\hat{E}_t$ is defined as that level of emissions for which the long run per annum (when capital is costlessly variable) and short run per annum (when capital is fixed) costs of holding emissions at $E_t$ coincide, given the composition of the capital stock at year *t*. Thus in Figure 1, the short and long run marginal cost of emissions reduction from the zero abatement level $\bar{E}(1+a)^t$, coincide at $\bar{E}(1+a)^t - \hat{E}_t$. From Le Chatelier's principle, the short run marginal cost of producing at any other level of emissions than $\hat{E}_t$ must exceed the long run marginal cost.

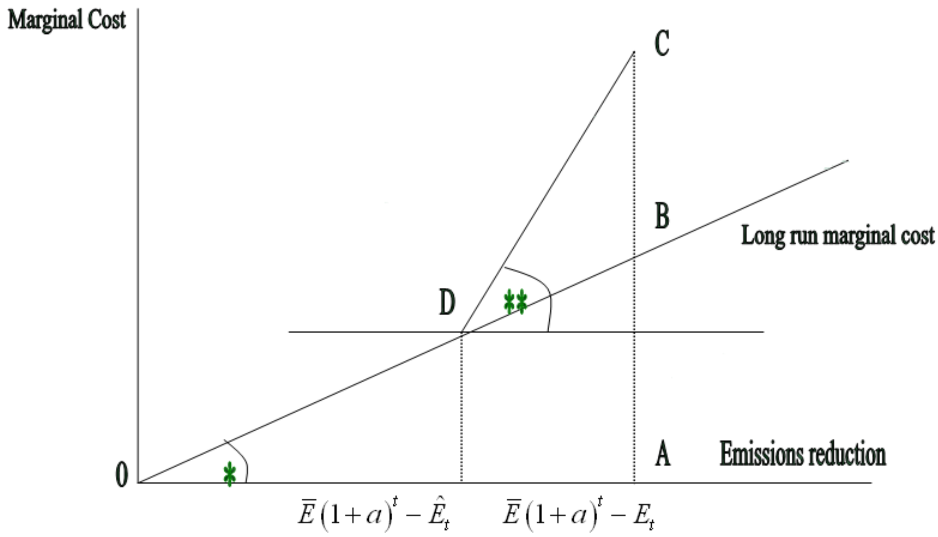


**Figure 1.** Short and Long Marginal Cost of Emissions Reduction for a Given Capital Stock Composition, where * = $g(1+a)^t$, ** = $\frac{1}{1-q} g(1+a)^{-t}$

Since estimates of the short run marginal costs from $CO_2$ are not available, they will be derived here from the long run marginal cost curve. It is assumed that in the long run, a fraction *q* of any unit change in emissions will come from changing factors fixed in the short run, and fraction 1-*q* from variable factors. Therefore the slope of the short run marginal cost from emissions reduction in Figure 1 is $\frac{1}{1-q} g(1+a)^{-t}$, which is vertical if *q* = 1, and coincides with the long run marginal cost when *q* = 0. The total short run costs of reducing emissions by $\bar{E}(1+a)^t - E_t$ in Figure 1 is triangle *0AB* (long run cost) plus triangle *DCB* (the additional cost because $\hat{E}_t$ is not at its most efficient level for producing $E_t$), which is

$$\frac{g(1+a)^{-t}}{2}\left[E(1+a)^t - E_t\right]^2 + \frac{g'(1+a)^{-t}}{2}\left[\hat{E}_t - E_t\right]^2 \qquad (12)$$

where $g' = \frac{qg}{(1-q)}$.

Triangle *DCB* will be referred to as "disequilibrium" costs.

For analytical convenience, the adjustment costs function for changes in $\hat{E}_t$ is assumed to be quadratic, and given by:

$$\frac{\eta}{2}\left[\hat{E}_t - \hat{E}_{t+1}\right]^2, \quad \eta > 0 \tag{13}$$

where $\eta$ is the slope of the marginal adjustment cost function. For an interpretation, suppose that capital in the $CO_2$ sector consists of identical machines which all have the same length of life *n* years, and there is fraction $\frac{1}{n}$ of the capital stock in each age group. If $\frac{1}{n}$ of the machine is to be scrapped at the end of a period, adjustment costs are zero since this just comes from not replacing machines becoming obsolete. If fraction $\frac{s}{n}$ is scrapped (where *s* is an integer and $s \leq n$) then machines with one years returns, two years returns, etc, up to *s* – 1 years returns will be scrapped and the loss of future returns per unit of capital, assuming no discounting, is

$$\frac{r}{n}\left[1+2+\ldots s-1\right] = \frac{rs}{2n}(s+1) \tag{14}$$

where r is the per period return on the total capital stock. This expression is the total adjustment cost per unit of capital in a period, which is quadratic in *s*, the amount of adjustment. Two reasons why the slope of the marginal adjustment cost function may be declining, rather than linear as imposed in (13) are (a) when allowance is made for discounting and (b) if the capital stock had previously been expanding over time, the proportion of young relative to old capital would be greater.

The present value of disequilibrium costs and direct adjustment costs, given a path for $\hat{E}_t$ and using a discount rate of *v* as before, is therefore, from (12) and (13),

$$\sum_{t=0}^{\infty} \frac{1}{(1+v)} t\left[\frac{g'}{2(1+a)} t\left(\hat{E}_t - E_t\right)^2 + \frac{\eta}{2}\left(\hat{E}_t - \hat{E}_{t+1}\right)^2\right] \tag{15}$$

The imposed emissions reduction path is given by

$$E_t = \frac{E_0 - E^*}{(1+z)^t} + E^* \qquad \text{where } 0 < z < 1. \tag{16}$$

which is chosen for analytical convenience. If a concave path were assumed, the optimal adjustment of $\hat{E}_t$, and hence the formula for cumulative adjustment costs, is extremely complicated: typically $\hat{E}_t$ is declining slowly at first, then more rapidly, then more slowly as it asymptotes towards the steady state. A further assumption is that the slope of the short run marginal cost function remains fixed at *g'*, rather than declining at rate a over time. This will lead to some overestimation of the cumulative adjustment costs, but greatly simplifies the formulas derived below.

The initial capital stock is assumed to be the one which maximizes long run efficiency, given emissions of $\overline{E}$ in year observed, hence

$$\hat{E}_0 - \overline{E} \tag{17}$$

Maximizing (15), setting $\alpha$ = 0, with respect to $\left(\hat{E}_{t+1}\right)_0^{\infty}$ and subject to (16), and (17) gives

$$-\frac{\eta}{1+v}\hat{E}_{t+2} + \left[\frac{g'}{1+v} + \left(1+\frac{1}{1+v}\right)\eta\right]\hat{E}_{t+1} - \eta\hat{E}_t - \frac{g'}{1+v}E_{t+1} \tag{18}$$

which has a steady state $\hat{E}^* - E^*$. Solving (18), dividing equation (16) by $-\frac{\eta}{(1+v)^{-1}}$ gives

$$\hat{E}_{t+2} - b\hat{E}_{t+1} + c\hat{E}_t = -\frac{g'}{\eta}E_{t+1}$$

where $b = \left[2 + \frac{g}{\eta} + v\right] > 1; c = 1 + v > 1$

using the lag Operator $L$, where $Lx_t = x_{t-1}$ gives

$$(1-\delta L)(1-\lambda L)\hat{E}_{t+2} = -\frac{g'}{\eta}E_{t+2} \tag{19}$$

where $\lambda\delta = c$ and $\lambda + \delta = b$. Therefore

$$\lambda = \frac{1}{2}\left\{\frac{1}{1+v}\left[2+\frac{g}{\eta}+v\right] + \left\{\left[\frac{1}{1+v}\right]^2\left[2+\frac{g}{\eta}+v\right]^2 - \frac{4}{1+v}\right\}^{\frac{1}{2}}\right\}$$

$$\delta = \frac{1}{2}\left\{\frac{1}{1+v}\left[2+\frac{g}{\eta}+v\right] - \left\{\left[\frac{1}{1+v}\right]^2\left[2+\frac{g}{\eta}+v\right]^2 - \frac{4}{1+v}\right\}^{\frac{1}{2}}\right\}$$

where $0 < \delta < 1$ and $\lambda > 1$. Dividing (19) by $-\lambda L$ gives

$$(1-\delta L)\left[1-\frac{L^{-1}}{\lambda}\right]\hat{E}_{t+1} = \frac{g'}{\alpha\lambda}E_{t+1}$$

where $L^{-1}x_t = x_{t+1}$. Dividing by $1-\frac{L^{-1}}{\lambda}$, integrating forward and substituting $E_{t+i} = \frac{(E_0 - E^*)}{(1+z)^{t+i}} + E^*$ gives

$$(1-\delta L)\hat{E}_{t+1} = \frac{g'}{\eta}\left[\frac{E^*}{\lambda-1} + \frac{E_0 - E^*}{(1+z)^t\left[\lambda(1+z)-1\right]}\right]$$

Dividing by $(1-\delta L)$

$$\hat{E}_{t+1} = \frac{g'}{\eta}\left[\frac{E^*}{(\lambda-1)(1-\delta)} + \frac{E_0 - E^*}{(1+z)^t\left[\lambda(1+z)-1\right]}\left[\frac{1}{1-\frac{\delta}{1+z}}\right]\right]$$

$$\therefore \quad \hat{E}_0 = \frac{g'}{\eta}\left[\frac{E^*}{(\lambda-1)(1-\delta)} + \frac{E_0 - E^*}{(1+z)^{-1}\left[\lambda(1+z)-1\right]}\left[\frac{1}{1-\frac{\delta}{1+z}}\right]\right]$$

$$\therefore \quad \begin{aligned}\hat{E}_{t+1} &= \frac{\hat{E}_0}{(1+z)^{t+1}} + \left[1-\frac{1}{(1+z)^{t+1}}\right]\frac{g'E^*}{\alpha(\lambda-1)(1-\delta)} \\ &= \frac{\hat{E}_0}{(1+z)^{t+1}} + \left[1-\frac{1}{(1+z)^{t+1}}\right]E^*\end{aligned}$$

gives

$$\hat{E}_{t+1} = E^* + \frac{\hat{E}_0 - E^*}{(1+z)^{t+1}}$$

This simple path for $\hat{E}_{t+1}$, which converges at $z$, results from the linearity of each marginal cost function. From (16) and (18)

$$\hat{E}_t - E_t = \frac{\bar{E} - E_0}{(1+z)^t} \tag{20}$$

and

$$\hat{E}_t - \hat{E}_{t+1} = \frac{z\left(\bar{E} - E^*\right)}{(1+z)^t} \tag{21}$$

Substituting (20) and (21) in (14), (with $\alpha$ = 0) gives

$$\frac{\omega\left\{\frac{g'}{2}\left(\bar{E}-E_0\right)^2+\frac{\eta}{2}\left[z\left(\bar{E}-E^*\right)\right]^2\right\}}{\omega-1} \qquad (22)$$

where $\omega=\left(1+v\right)\left(1+z\right)^2>1$

which is the present value of disequilibrium and adjustment costs, arising because capital is not costlessly variable in the short run, since (a) $\left(\bar{E}-E_0\right)^2\omega\left(\omega-1\right)^{-1}$ is the present value of the gap between actual emissions and the cost minimizing level over time and $\frac{g'}{2}$ is the cost per unit of this difference as of year observed. (b) $\left[z\left(\bar{E}-E^*\right)\right]^2\omega\left(\omega-1\right)^{-1}$ is the present value of adjustments in the capital stock, and $\frac{\eta}{2}$ is the cost per unit of adjustment.

## 3. Conclusion

The results of this paper suggest that the welfare cost of having to take a lot of action to reduce CO2 emissions in the future, should the problem of global warming turn out be very serious. A drawback of using the present value of utility as a measure of welfare is that it can obscure substantial transitory income losses due to adjustment costs. Another potentially important problem is that the transactions costs of getting a large group of countries to agree on, and enforce, an emissions reduction agreement, are ignored.

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