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Nonlinear Least Squares (NLS) Tracer Model

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ABSTRACT

Tracer test is used to study possible temperature decline and cooling of production well during long-term re-injection through studying connection between injection and production well. Tracer test involve injecting a chemical tracer into a hydrological system and monitoring it's recovery through time. Concentration tracer in production well is nonlinear model. Nonlinear Least Squares Method is proposed to estimate the tracer parameter, i.e. average fluid velocity in the channel and dispersion coefficient, by using simulation data tracer concentration from production wells AH-4bis, AH-19, and AH-22 in Ahuacapan, Elsavador. The results of estimation are used for predicting thermal breakthrough and cooling production well during long term re-injection.

Keywords: tracer test, tracer velocity, dispersion coefficient, NLS.

1. Introduction

Geothermal re-injection is being used to maintain reservoir pressure and to extract more thermal energy from reservoir rock[1]. The possible cooling of production wells, or thermal breakthrough is one of the main disadvantages associated with injection. For that reason, tracer test is done to study effect of re-injection through studying connection between injection and production well. Tracer test involve injecting a chemical tracer into a hydrological system and monitoring it's recovery through time[1].

This paper proposes an estimation methodology for the average fluid velocity in the channel and dispersion coefficient using Nonlinear Least Squares Method because the Concentration tracer in production well is nonlinear model. The results of estimation are used for predicting thermal breakthrough and cooling production well during long term re-injection.

2. Tracer Model

A simple one-dimensional flow-channel tracer transport model has turned out to be quite powerful in simulating return data from tracer test[1]. It assumes the flow between injection and production well may be approximate by one-dimensional flow in flow-channels, as shown in figure 2.1. In the case of one-dimensional flow, the equation is

$$D\frac{\partial^2 C}{\partial x^2} = u\frac{\partial C}{\partial x} + \frac{\partial C}{\partial t} \qquad (2.1)$$

Where D is dispersion coefficient (m/s^2) , C is the tracer concentration in the flow channel(kg/m^3), x is the distance along the flow channel and u is the average fluid velocity in the channel. $u = q/\rho A\phi$, where q is the injection rate (kg/s), ρ the water density (kg/m^3), A is the average cross-sectional area of the flow channel (m^2) and ϕ is the flow-channel porosity.

When t=0, the tracer is injected M(kg) with the high injection rate q, the solution of equation 2.1 simplifies to

$$c(x,t) = \frac{uM}{Q}\frac{1}{2\sqrt{\pi Dt}}e^{-(x-ut)^2/4Dt} \qquad (2.2)$$

Where c(x,t) is actually the tracer concentration in the production well, Q is the production rate (kg/s) and x is the distance between the wells involved. The equation (2.2) according to $c.Q = C.q$ has been assumed.

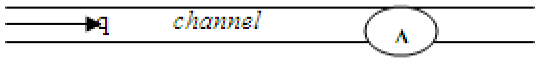


Figure 2.1 A Schematic Figure of a Flow-Channel
with One-Dimensional Flow Connecting an Injection Well and a Production Well

3. Nonlinier Least Square (NLS)

Tracer concentration tracer model in production well is nonlinear model. To estimate the parameter tracer is being used to nonlinear least square method[2]. The equation 2.2 is written as

$$f(\boldsymbol{\xi}_\mathbf{u}, \boldsymbol{\theta}) = \frac{\theta_1 M}{Q} \frac{1}{2\sqrt{\pi \theta_2 \xi_u}} e^{-(x-\theta_1 \xi_u)^2 / 4\theta_2 \xi_u} \tag{3.1}$$

Where $\boldsymbol{\xi}_\mathbf{u}$ =($\xi_1, \xi_2, \xi_3, ..., \xi_k$) is *predictor variable*, and $\boldsymbol{\theta} = (\theta_1, \theta_2)$ is the parameter *u* and *D*.

$$Y_u = f(\boldsymbol{\xi}_\mathbf{u}, \boldsymbol{\theta}) + \in_u \tag{3.2}$$

The error sum of squares is

$$\begin{aligned} S(\boldsymbol{\theta}) &= \sum_{u=1}^{n} \{Y_u - f(\boldsymbol{\xi}_\mathbf{u}, \boldsymbol{\theta})\}^2 \\ &= \sum_{u=1}^{n} \{Y_u - \frac{\theta_1 M}{Q} \frac{1}{2\sqrt{\pi \theta_2 \xi_u}} e^{-(x-\theta_1 \xi_u)^2 / 4\theta_2 \xi_u} \}^2 \end{aligned} \tag{3.3}$$

To get least square estimate of $\hat{\boldsymbol{\theta}}$, the first derivative of the Equation (2.5) to $\boldsymbol{\theta}$ is zero[3]. A Taylor linearization is used to obtain a solution for $\hat{\theta}$.

$$f(\boldsymbol{\xi}_\mathbf{u}, \boldsymbol{\theta}) = f(\boldsymbol{\xi}_\mathbf{u}, \boldsymbol{\theta}_\mathbf{0}) + \sum_{i=1}^{2} \left[\frac{\partial f(\boldsymbol{\xi}_\mathbf{u}, \boldsymbol{\theta})}{\partial \theta_i} \right]_{\boldsymbol{\theta} = \boldsymbol{\theta}_\mathbf{0}} (\theta_i - \theta_{i0}) \text{ where } \boldsymbol{\theta}_\mathbf{0} = (\theta_{10}, \theta_{20}, ..., \theta_{p0})'.$$

4. Simulation

Computer software *S-Plus* can be used to simulate tracer data from production well and to estimate tracer parameter. The simulation based on data from production well AH-4bis, AH-19,AH-22 which is presented in table 3.1.

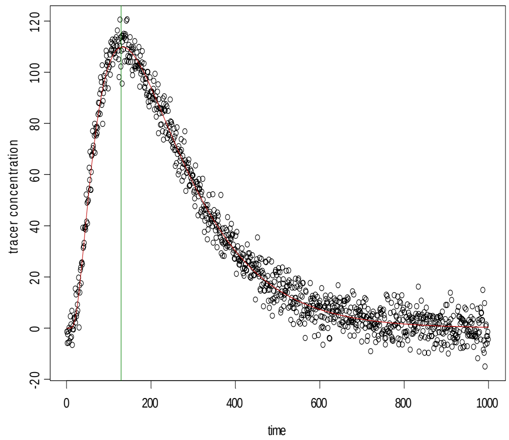
Table 3.1 The Tracer Parameter from
Production Well in Ahuacapan, Elsavador[1]

	AH-4bis	AH-19	AH-22
x(cm)	800	300	600
q(kg/s)	64	40	20
Q(kg/s)	31	7	7
xA ϕ (m^3)	6300	5300	4200
α_L (m)	240	40	150
u(m/s)	8,127	2,264	2,857
D(m^3)	1950,47	90,566	428,571

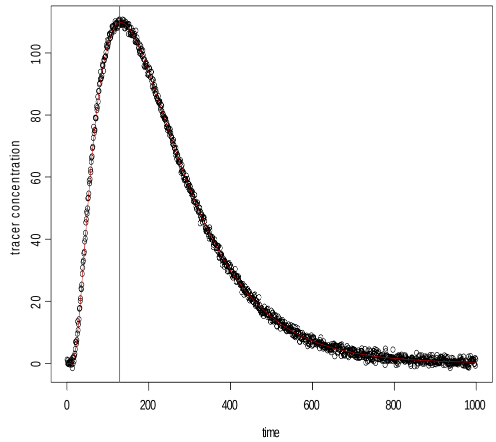
Listed below is an example program in S Plus to simulate tracer data and estimate tracer parameter[3]. The data based on AH-4 bis production well, where the noise variance is 0,2.

```
x<-cbind(1:1000)
y<-exp(-((800-8.127*x)^2)/(4*1950.476*x))*(8.127/31)*(1/2*sqrt(3.14*1950.476*x))
y<-y+rnorm(1000,0,0.2)
our.exp<-data.frame(x=x,y=y)
our.exp
plot(x,y,xlab="t",ylab="Konsentrasi Tracer")
fit1<-nls(y~exp(-((800-
    u*x)^2)/(4*D*x))*(u/31)*(1/2*sqrt(3.14*D*x)),start=(list(u=9,D=1000)),data=our.exp)
summary(fit1)
fit1$param
fit1$resid
fit1$fitted
max(y)
plot(x,y,xlab="waktu",ylab="konsentrasi tracer")
title("regresi konsentrasi tracer")
lines(x,fitted(fit1),col=8)
abline(v=129,col=4)
```

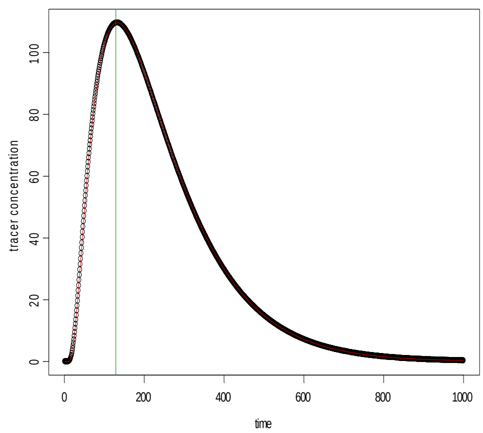
The results of simulation are presented in table 3.2 and figure 1



(a)



(b)

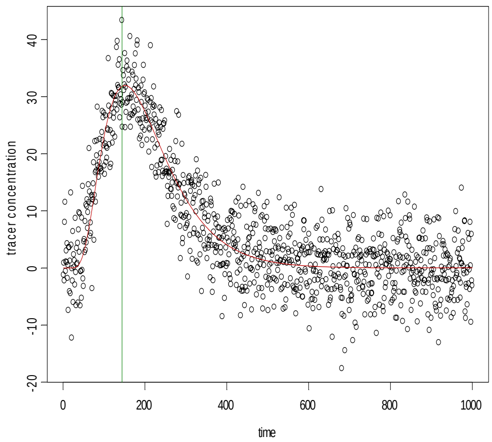


(c)

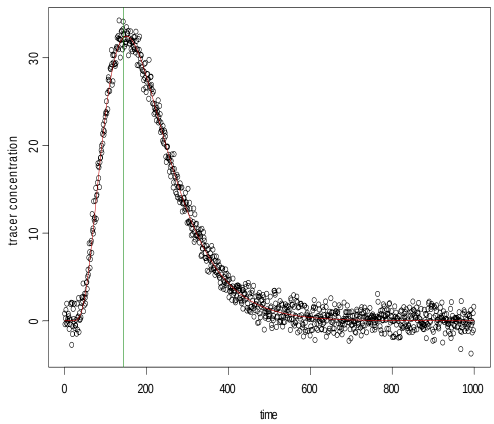
Figure 1. NLS Curve of Tracer Concentration
from AH-4bis with Noise Variance: (a) 5, (b) 1, (c) 0.001

Table 3.2 The Estimation of u and D Value
AH-4bis Production Well, with Initial Value θ_1 =9 and θ_2 =1000.

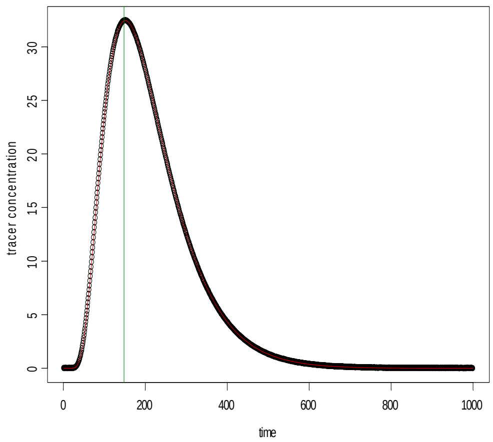
θ_1	θ_2	SS(θ)	Noise Variance
8,153	1967,26	5,071	5
8,127	1949,42	1,002	1
8,127	1950,5	0.010	0,01



(a)



(b)



(c)

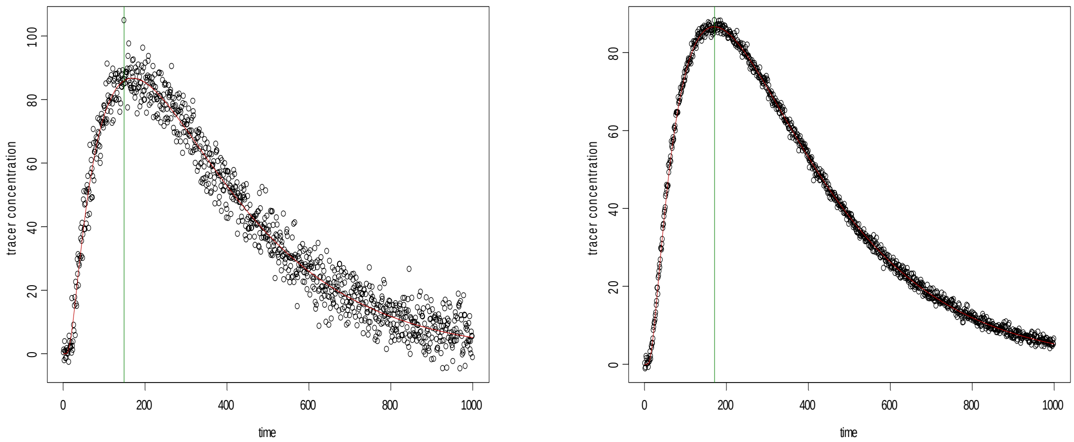
Figure 2. NLS Curve of Tracer Concentration from AH-19, with Noise Variance : (a) 5, (b) 1, (c) 0.001

Table 3.3 The Estimation of u and D Value
from AH-19 Production Well, with Initial Value θ_1 =3 and θ_2 =100.

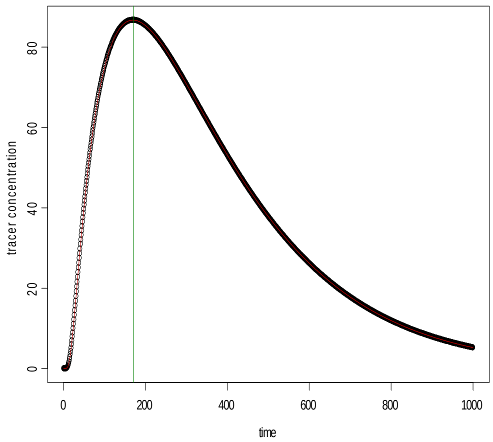
θ_1	θ_2	SS(θ)	Noise variance
2,277	93,02	5,069	5
2,262	90,38	1,016	1
2.264	90.57	0.001	0,001

Table 3.4 The Estimation of u and D Value
from AH-22 Production Well, with Initial Value u=3 and D=100

θ_1	θ_2	SS(θ)	Noise Variance
2.856	427.95	4.933	5
2.857	428.63	0.982	1
2.857	428.57	0.001	0,001



(a) (b)



(c)

Figure 3. NLS Curve of Tracer Concentration
from AH-22, with Noise Variance : (g) 5, (h) 1, (i) 0.001

In figure 3.1, 3.2, and 3.3, red is denote nls curve, black denote simulation curve, and green line denote the point of maximum value of tracer concentration. The smaller of error going to make nls curve close up to simulation curve. It's mean that the value of estimation approach to the value of simulation.

5. Conclusion

Nonlinear Least Squares method is a suitable method to estimate tracer parameter because the nonlinearity of tracer equation. As the variance of the noise is close to zero, the estimator is close to the value of the parameters, as expected therefore, the proposed approach gives a reasonable results.

Reference

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